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Journal
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The Franklin Institute

1826

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Strengthened Science Education as a National Goal

JAMES R. KILLIAN, JR.

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Creep Collapse of Cylindrical Shells

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VOL. 272 NO. 1

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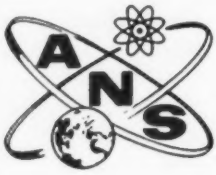

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The American Nuclear Society, an organization of individual scientists and engineers, was founded on October 11, 1954, in Washington, D. C., by a group of men active in nuclear science and technology. The main objectives of the Society are the advancement of nuclear science and engineering, of allied sciences and arts, and the integration of the several disciplines constituting nuclear science and technology. Other objectives include the encouragement of research, the establishment of scholarships, the dissemination of technical information, the holding of meetings devoted to scientific and technical papers, and cooperation with government agencies, educational institutions and other organizations having similar purposes.

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STRENGTHENED SCIENCE EDUCATION AS A NATIONAL GOAL *

BY

JAMES R. KILLIAN, JR.[1]

My first desire in addressing this meeting is to salute The Franklin Institute for its bold and well-conceived Program to Advance Science Education. This is an admirable undertaking in every respect, one that will enhance the economic and cultural resources of this community while at the same time bringing new opportunities for individual fulfillment to people of all ages, but especially the promising young people in the schools.

What is being done here at the Institute to advance science education warrants the sustained and enthusiastic support of the citizens of this great city and region, for it stands as one of the most important and imaginative efforts in the nation to enhance, through education, the national strength in science.

I speak tonight of the social and cultural importance of science and engineering in our kind of industrial society and of how this importance imposes heightened requirements for excellence and balance in our education.

In directing my remarks toward this broader role of science and engineering, I do not for a moment play down their importance to our national security. The quality of our military technology is of the utmost importance to our safety, and we permit it to become second best only at the risk of national disaster. Our nation is in grave danger, and we dare not let down our guard.

* Delivered on April 19, 1961, in Franklin Memorial Hall at The Franklin Institute, before a distinguished audience of Delaware Valley industrialists, educators, and civic leaders.

[1] Chairman of the Corporation of the Massachusetts Institute of Technology, Cambridge, Massachusetts; and Chairman of the President's Foreign Intelligence Advisory Board.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

But neither can we fail to strive for peak performance in every part of our national life. The national security depends not alone on military and economic strength. It depends also on the quality of our society, the degree to which it inspires and expresses the highest aims of mankind, and the measure of its success in cultivating its human resources and in releasing the latent capacity of each for his individual fulfillment. These factors bearing upon the excellence, the taste, the benignity, and the humaneness of our society are vital to our national strength. I speak of the ways in which the scientist and engineer can contribute to these qualities, of how their education affects their capacity to do so, and of the importance of scientific literacy for all citizens if they are to contribute most to the achievement of a great society in an age of science. I speak, too, of the growing role of the scientist and engineer—and the institutions which educate them—in international affairs and in the preservation and extension of our Western heritage.

Those of us who have served in recent years as presidential advisors and in relating science and technology to top-level policy making in Washington and who have worked strenuously to mobilize science for national security have each found ourselves becoming steadily more convinced that science, while continuing to serve national security with a sense of urgency, ought also concurrently to mount a greater effort in those non-military activities which can add strength to the Free World. It should mount a greater effort to augment its contribution to international affairs, to the formulation of arms limitation policies, to promote economic growth, domestic and foreign, and to widen man's view of himself and of the world. In the words of President Kennedy, let us invoke the wonders of science instead of its terrors. Let us expand our efforts to explore the stars, to conquer the deserts, to eradicate disease, to tap the ocean depths, to increase commerce.

Let me speak first of the changing role of the scientist and engineer in domestic affairs. I do not dwell on their fundamental work in advancing science and engineering. It is, of course, their first great responsibility to be creative masters of their professional work and to preserve its integrity. The Committee on Science in the Promotion of Human Welfare of the American Association for the Advancement of Science said it well when it observed:

> As science becomes more deeply involved in the frequently discordant affairs of public life and in highly competitive social endeavors, we may expect a growing pressure toward relaxation of the traditional rules for the conduct of science: objective, open communication of results; rigorous distinction between fact and hypothesis; candid recognition of assumptions and sources of error. It is these rules which permit science to progressively increase our knowledge of and control over nature. Without them science becomes useless, and even dangerous to the social order.

I speak not only of these basic responsibilities but rather of the growing role of the scientist and engineer "to help mediate" the effects of scientific progress on human welfare.

Of top priority on the agendas of science and education is the importance of making the true values of science more widely understood as an expression of our culture. Today the picture of the scientist in the public mind is a confused triple exposure which includes the images of the scientist, the engineer, and the technician. For the benefit of all three, and to clarify the liberalizing content of science in general education, the images need to be separated. Science is not space vehicles, nor rockets, nor miracle drugs, nor nuclear weapons, nor automation, nor gadgets. All of these are the products of technology made possible by science, but they are not science.

In the Report of the President's Commission on National Goals, Warren Weaver eloquently said what science is:

> The primary concern of science is the understanding of nature—of our physical environment and the world of living things, including man himself. . . .
>
> For example, we are just in the process of gaining a scientific picture of the total ascent of life. By far more vast and significant than the Darwinian view, this modern evolutionary doctrine begins with the elementary particles of the nuclear physicist and moves through the whole range of the atomic and molecular world up to the nucleic acids which, in their capacity to reproduce pattern and to pass on coded information, seem capable of forming the primitive basis for a living organism. From this point it is conceivable to move on to the gene, the chromosome, the cell, and ultimately human life. Whether or not man is the present climax of this ascent is itself now under question: for we have radar-listening devices, directed at inconceivably distant parts of the cosmos, seeking to determine whether there are other and possibly more advanced beings there, trying to communicate with earth-bound man.
>
> When the sights are set as high as this, the view transcends all the compartments of science. This is not, in any exclusive sense, physics or biology or chemistry or astronomy. This is the whole of science, engaged with a problem of majestic dimensions.
>
> The sweep and the depth of such a view of matter, man, and the universe fairly suggest what science really is—not a trivial business of tricky hardware, not the phony bubbling retorts of the advertisements, not strange men with white coats or beards, but the response, at once poetic and analytical, of man's creative mind to the challenge of the mystery of matter and life.

It is science in this sense that we need to understand and to emphasize as an integral part of our culture. Viewing and understanding science in this way, modern man in the twentieth century has a better chance to make this a better world, to carry his vision to the awakening

peoples, and with them to gain new confidence in the benignity of intelligence, and its capacity to build a better world.

This view of science, and the effort to advance this kind of science, is a better view and objective for a nation than the extravagant, prestige-motivated exploitation of "technological spectaculars" as a way of displaying its scientific leadership. The image of a nation may be shaped and made impressive to the world more by the quality of its inner life than by its exploits, however valuable, with prematurely projected nuclear propelled aircraft (now happily grounded), lunar landings, premature nuclear power plants, or other glamorous enterprises.

Next, I emphasize the current shortage of scientists and engineers who can fulfill the responsibilities of public office and serve as science administrators. We do not have enough competent scientists and engineers to fill the many important posts in government, particularly at the policy level, requiring scientific and engineering training. Many important Washington posts are inadequately manned today, and some are vacant, because scientists and engineers of adequate scope and competence are not available or are unwilling to accept appointment to them.

We do not have enough scientists and engineers who have a deep understanding of their specialty but who have, in addition, the cast of mind, the motivation, and the breadth of understanding to serve effectively—and to survive—in policy-making, advisory and administrative roles in the public service.

We have hardly any scientists and engineers in politics. There should be scientists and engineers in Congress as there are now beginning to be in state legislatures and in other elective offices. Government in a technological society requires a reasonable complement of scientists in the public arena if it is to deal wisely with all the great policy matters arising out of science.

I stress next the broader responsibilities vested in the engineering profession.

Even though scientist and engineer grow closer in knowledge, method, and work, and their domains increasingly overlap, the role of the engineer remains distinct and importantly differentiated. He is the liaison between the laboratory and the layman, between theory and practice, between things and people. He is concerned with research, but also with costs, with schedules, with the management of people. He provides the scientists with the instruments they require to advance science. He applies the concepts and discoveries of the scientists for the practical needs of the human community, its health, prosperity, and individual fulfillment.

Because science is advancing so rapidly today, never has the engineer had so much new and complex knowledge to master, such a cornucopia of resources at his command, such requirements of adaptability to meet.

More and more, industry is drawing upon its engineers to fill posts in top management, and many of our companies, especially those dependent upon advanced technology, are headed by men with engineering education and experience. I believe that industry increasingly must look to the engineering profession for such top leadership.

As industry looks to engineers for management competence, it has a continuing problem in providing adequate rewards to its scientists and engineers for their performance *as scientists and engineers*. Too often they find this status unsatisfactory and seek to become managers or salesmen, where they feel rewards and status may be greater.

While scientists and engineers quite properly should always be considered a pool of talent from which to recruit managers, something is obviously wrong when environmental factors and values lead a high percentage of research engineers and scientists to be dissatisfied with sustained careers in research or in engineering. This is a condition harmful to industry and to the quality of industrial technology and one that must command more creative attention on the part of management.

Another responsibility resting upon the engineer is to give increasing attention to ways in which the engineer can improve the quality of our society by his attention to non-utilitarian values as well as utilitarian. Today the engineer faces an array of challenges arising out of population growth, greater urban concentration, and the intensification of industry. We face an acceleration in the decay of cities, the menace of pollutants and contaminants, the need for conservation, and the struggle against ugliness and bad taste. His willingness to meet these challenges, to rise above narrow utilitarian considerations, will be a measure of the engineer's capacity to shape technology to serve the quality as well as the material needs of our life. As one of my humanist friends has remarked, the great opportunity for engineering today is to help humans to be more human.

Every profession and every individual has a responsibility to combat ugliness and deterioration in modern industrial societies, but I venture to suggest that engineers, especially civil engineers, by working closely with city planners and architects, are in a key position to mobilize the forces required to achieve a new level of humaneness and benignity in our environment.

This is true in terms of individual structures as well as in terms of city and regional plans. In many of the great bridges and engineering structures they have designed, engineers of today have achieved some of the most beautiful structures of our time, but there are not enough of them and too many that are ugly. We need more structures possessing the beauty achieved by the late Robert Maillart, and by Torroja, Freysinnet and Nervi. I am not ready to accept as adequate the explanation that high labor costs are the principal reason why the thrill-

ingly beautiful structures of these gifted engineers have been so infrequently matched.

Ugly engineering structures, however practical, may in the long run be an economic waste. Worse, they do damage to the human spirit.

I come now to the increasingly complex managerial responsibilities of the engineer. One of the rarest types of technical management talent is that which comprehends the whole array of requirements, both human and technological, which enter into the building of great engineering systems—a new power plant, a data-processing system, a nuclear submarine, a satellite communication network. Men are in very short supply who can coordinate successfully the elements of research, development, test and evaluation, and production for these intricate systems. First-rate component development is not sufficient; there must be program managers who comprehend, with loving attention to detail, every element of the system, but who also can master the systems integration and the organizational and personnel problems involved in planning and administration.

There are too few engineers in the world today who have this mastery of systems development. When industry has failures in meeting performance requirements, or schedules, or budgets for intricate products, we can usually trace them to weakness in this kind of engineering management. As technology grows more complex and as the rate of technological obsolescence quickens, this kind of engineer-manager will come to occupy a still more crucial position in technological societies.

In governments and industry the need steadily grows for talent that can make decisions about technical feasibility, the kind of technical judgment and foresight that can avoid plans and requirements which lead to unnecessary engineering complexity, or programs not readily attainable within existing technology, or undertakings which will be obsolete before completed.

These concepts are also related to increasing industrial productivity. Rising productivity today depends as never before on the application of an advancing technology to the productive process.

The engineer must also share in the management responsibility of cushioning the social effects of the technological change he produces. However deeply he may believe (as I do) that automation is of great economic and social advantage, he must face the fact that it can cause local or temporary displacements in employment and that he must accept responsibility to minimize or to avoid these effects; he cannot, without being socially irresponsible, ignore them. Careful, inventive planning by joint management-labor-engineering groups can do much to insure an orderly management of technological innovation without slowing down the rate of innovation or dampening the spirit of discovery and improvement.

As one example of his widening role, the engineer in this era of

awakening nations has an unprecedented opportunity to work sensitively in strange cultures to give them in their own terms the technology they need and can use to become self-sustaining.

In all of these ways he performs a unique service for the human community. Never has he had so great an opportunity to use his professional insights and skills for great and noble professional and social purposes.

Today scientists and engineers have a new role in the international arena. They are in a uniquely favorable position to help in reducing tensions, in encouraging cooperation between nations, and in meeting the crisis of the newly developing nations. I do not suggest that science is the sovereign remedy for the world's turmoil, strife, and cleavages. It is not. Political leadership marked by wisdom, imagination, and maturity is an overriding requirement, and all the professional resources and wit of nations are required to keep the peace and to maintain the Free World strong, free, and unified. Trade and credit and economic organization can profoundly affect the unit of the Free World. Without neglecting any of these factors, I nevertheless stress that at this time there is a tide—a scientific tide—in the affairs of men, which taken at the flood, may lead on to fortune.

We see today a diverse array of international relationships and communities of interest flowing over national frontiers and free of the restraints and inhibitions which must shape the official relationships between governments. This informal network of cultural contacts—especially the discourse among scientists and engineers in the world's commonwealth of science—is serving, however modestly, to better communication and understanding. The surging flood of scientific literature penetrating every part of the world is one example; the series of "Pugwash" conferences is another; the Rehovoth Conference on Science in the Advancement of New States, held in Israel last summer, still another. I know one colleague who came back from Rehovoth fired with a new vision of what might be done for African education.

As often mentioned, the 1955 Conference on the Peaceful Uses of Atomic Energy was a breakthrough in reopening the dialogue among scientists and engineers from East and West. That great meeting demonstrated anew what Herman Melville meant when he wrote that "genius all over the world stands hand in hand, and one shock of recognition runs the whole circle around." It established once again cultural ties that provide hope that indeed it is still possible to lessen tension and suspicion between East and West. It and other conferences which have followed have shown, as I. I. Rabi has remarked, "that science and peace do have a connection, not yet as great as science and war, but certainly not negligible."

The International Geophysical Year is the most impressive example of successful cooperation among nations with deep ideological differ-

ences and of what can be accomplished by voluntary activities initiated by private organizations. In the recent Antarctica Treaty we have an example of what such private, voluntary action can help to achieve. The International Geophysical Year made a major contribution to the creation of this concept of an international laboratory area and to the realization of a major diplomatic advance in peaceful cooperation among nations. As a result of this twelve-nation treaty, territorial claims are not pressed in Antarctica; there is free access to all parts of the area, thus providing a truly international laboratory for scientific work, and the participating nations have provided for a true and inspectable demilitarized zone. As General Eisenhower once suggested, the Antarctica Treaty may also be a prototype for a treaty establishing the peaceful use of outer space.

And now, in a fashion similar to the International Geophysical Year, scientists from a number of countries are shaping a plan for a coordinated oceanographic study of the Indian Ocean, and a plan is afoot for a global resurvey of the Earth's magnetic field, and proposals for an International Medical Year is unfinished business on the UN agenda.

These are all important. Their contribution is so great that similar efforts to coordinate programs should be vigorously encouraged. But they are not enough. More coordination is not enough. Through bilateral and multilateral arrangements we need more cooperative projects which bring nations together as partners in working enterprises. We need more projects like CERN—the European Center for Nuclear Research

A host of such cooperative projects have been proposed, especially in space, agriculture, and medicine, but disappointingly few have been realized. Governments tend to become timid in the face of actual undertakings. Let us hope that the record improves, because the stakes are high; in the countries involved, including our own, there is a need for officers and agencies with the boldness, determination, and imagination to act.

I venture to emphasize one region where joint action to put science to work is critically needed: the Western Hemisphere. This area presents an immense opportunity, as President Kennedy recently emphasized, for good neighbors to work together in common scientific enterprises and to use technology to raise standards of living through the entire hemisphere. One need only point to the Institute of Nutrition of Central America and Panama to show an impressive example of what can be accomplished by international action, or to the Pan American Health Organization. It is vital to the peace and stability of the Free World that science and technology, including, of course, medicine and public health, really be mobilized for the benefit of the entire Western Hemisphere.

I. I. Rabi, who has made so many useful suggestions for the peaceful

uses of science, has proposed that we open one of our major American research centers to scientists from other lands, especially those from the Western Hemisphere. This proposal has great merit and I hope that a practical way might be found soon to put it into effect. Such an international laboratory could advance the progress of science and strengthen the ties which undergird the Western Hemisphere.

Since our principles and policies impel us to aid the new and less developed nations, we must face the fact that our knowledge of how to do it is frequently as underdeveloped as the nations we would help. Our aid programs should be supported and guided by a research program as intensively and professionally pursued as the research and development programs underlying our defense. An idealistic impulse to aid is not enough; we must bring together teams of specialists—economists, engineers, agriculturists, sociologists, educators—carefully prepared and armed with research results for work in the specific area to be aided.

The scientists and engineers of our generation face a magnificent challenge to join in interdisciplinary concert with the legal and political experts, and the social scientists in meeting this crisis of our relations with the awakening nations.

Clearly, this great domain of foreign affairs is one of the principal arenas where the scientist and engineer is sorely needed, where they have a new role that requires new attitudes, skills, and orientation, and where there is opportunity to make clear to all peoples that the science and technology of the Free World are profoundly committed to the survival, the welfare, and the dignity of all men.

What I have said so far is prologue to my plea for an unremitting effort to improve our science education in the United States. The expanding roles of the scientist and engineer call for an education to match their new duties and opportunities. In the most fundamental sense, the educational task is to broaden the content of education to include science as an essential element in the education of all citizens, to strengthen and expand our professional education of scientists and engineers, and to educate enough more scientists and engineers so that we will have enough not only to do the basic work of science and engineering and to meet the growing needs of education and industry but to have a margin over and above these needs from which may be drawn scientists and engineers for the public service, both domestic and foreign, and to provide an adequate number of science administrators.

The President's Commission on National Goals had basic things to say about education in general and science education in particular. For our total educational effort greater resources must be mobilized, and a higher proportion of the gross national product devoted to educational purposes. By 1970 our total of private and public expenditures for education must approximate 40 billion dollars, which is double the

1960 rate. We must spend 5 per cent of our gross national product for education, as against less than 4 per cent today.

We must make sure, the report emphasized, that students in high school, and even in the elementary grades, be appropriately exposed to the interest, importance, beauty, and excitement of science, and that every young person who has the desire and capacity to become a scientist can have available the best science education our leading scholars can provide. Given the availability of such education, science, the report continues, will find its fair share of the pool of talent, but this pool must be enlarged to the maximum.

These are all truisms that need reiteration but are there not specifics which can be recommended and stressed? Let me cite several by way of illustration, each representing important innovations in education, and each drawn from a wholly different level or aspect of education.

Let me first cite an innovation in high-school science-teaching—one which has been furthered in this locality by The Franklin Institute.

This year some 50,000 secondary school students in the United States are getting a coherent, modern fundamental introduction to physics by means of a new course prepared by a team approach—by the Physical Science Study Committee under the leadership of Professors Jerrold Zacharias and Francis Friedman. Motivated by a desire to make science really a part of our culture, this program represents a major advance in science teaching. It includes a whole new battery of interrelated teaching materials—textbook, movies, new laboratory equipment and newly designed experiments, and a wide range of collateral reading. The spirit and methods generating in this new program are now spreading, with the chemist, the biologist, and the mathematician all developing new courses, with first-rate scientists and first-rate teachers from universities and secondary schools joining in the work. Another group would like to work on grammar-school science, and now the American Association of Physics Teachers is initiating a similar effort to improve college physics. Altogether, what the Physical Science Study Committee has produced and inspired may well represent a revolutionary approach to upgrading, deepening, and enriching the substance, the taste, and the cultural content of science teaching. The implications of all this are profoundly important; yet the funds available to continue such efforts are exceedingly difficult to come by. In fact when one totals up the funds available or proposed for American education, the amounts included for this kind of basic improvement in content and teaching materials seem wholly inadequate. For a small increase in expenditures for this purpose, dramatic improvements can well be achieved in the quality of education.

I next cite a novel proposal for international education.

Last year a study group sponsored by the Science Committee of NATO published a report on "Increasing the Effectiveness of Western

Science." Among the many recommendations made by this group is that an international institute of science and technology should be established in Europe. This proposal has generated a surprising amount of enthusiasm. The NATO Parliamentarians Conference endorsed it heartily and a working group has been formed to examine the proposal in more detail.

To realize the full potential of science and engineering, Western societies must rapidly plan and provide for an enlarged and improved capacity of university education in these fields. The NATO report estimated that the number of engineers educated annually per million inhabitants is 400 for the Soviet Union, 200 for the United States, and 70 for the European Economic Community. Another report prepared under the auspices of the United Nations concluded the number of engineers in Europe should increase in the next decade by 3 to 4 per cent a year whereas existing educational capacity permits a growth of only 1 to 2 per cent. In the United States the President's Science Advisory Committee has twice recommended doubling the capacity of our graduate schools of science and engineering by 1970.

In both Europe and the United States the most acute shortage is of science teachers in both secondary schools and colleges. The United States faces a special problem in that the enrollment in its engineering schools has declined for four years. Some of this decline is accounted for by a shift of students from engineering into the sciences, mainly science and mathematics, but even so the decline in engineering enrollment, along with the decline in applications for medical schools, can only be viewed with grave concern.

The NATO report suggests the feasibility of regional action to advance science. Are there not other regions of the world ready for carrying through such international effort in science?

Let me comment on another significant development in education blending science and social science.

I speak of the appearance of new programs and centers in international studies and in political science. In at least three American universities new programs have been started dealing with science and public policy. The past fall term I had the stimulating experience of giving a graduate seminar in this subject and subsequently I have met with seminar groups in two other institutions. This experience has convinced me that this new field is one of importance, that there is a wealth of interesting material to be examined and codified, and that we need a continuing study of the impact of science on government and on foreign affairs and the innovation this impact has caused. More importantly, it has revealed the new and invigorating winds blowing in political science, and the cross-fertilization that is taking place between physical science and political science. It has recently been said that disciplines other than science and engineering may well begin to de-

velop a dynamic quality, a rapidity of change, not unlike science. I suggest that political science may be such a field. It has been remarked that the United States has not had a great political philosopher—a Locke or Hobbes—certainly not one since the Founding Fathers. Perhaps the interplay of science and politics may provide a new stimulus for a new creative surge in this field.

And here let me interpolate an observation about the use of competence groups in government. In 1957 the first science advisor to the President of the United States was appointed, and an independent science advisory committee reporting directly to the President was established. The format of these arrangements was such that they anticipated ways to reduce the dangers discussed by C. P. Snow in his Godkin Lectures on "Science and Government." The President's Science Advisory Committee can be independent of the Special Assistant to the President. It can report directly to the President if it chooses. It can elect its own chairman. It can bring to the Special Assistant a range of views and an objectivity that no single person could hope to achieve. And with its panel system it provides the policy-making agencies of government an opportunity to have roots deep in the creative community of science. All this, I suggest, was an important innovation in the use of competence groups by government, an invention that has been effectively used and relied upon by two Presidents.

And now let me conclude with four observations which seem to me to be relevant to any program for strengthening science and engineering and the education they require.

First, the quality of our science and our science education cannot be separated out from the quality of our intellectual life generally. What we are concerned with basically is the importance which the American people give to the factor of excellence in our society. It is basically important that we achieve a greater respect for learning, a greater pride in intellectual achievement, a willingness to assign education a higher place in our national list of priorities.

My next observation relates to the motivations in our society which give vigor to our science and technology and which are important to our continuing strength. So far we have demonstrated a sustained eagerness to find better ways of doing things. We have forged ahead because we wanted things to change. We have wanted to look forward and not backward. The revolution of modern man—the revolution which has found its fullest expression here in the United States—lies essentially in this. It is a revolt against things as they are when there are ways of doing things better. It is a revolution based upon determination rather than determinism. It is a revolution against all the forces which hinder man in building a better life. Science has had

a major part to play in shaping this basic American faith in creative change and improvement. We must continue this revolution.

My third observation has to do with the great responsibility which rests upon science today in the light of the extraordinary opportunities to participate in the formulation of national policy which it has been given. The growing linkage of science and technology with government demands of science a new order of poise, steadiness, and statesmanship. It demands of scientists who serve in advisory capacities a deep understanding of the role and the limitation of the advisor.

The current emphasis on science, if it is not to cause reactions adverse to science, also requires of the scientific community humility and a sense of proportion. It requires of scientists a recognition that science is but one of the great disciplines vital to our society and worthy of first-rate minds—a recognition that science is a partner sharing and shouldering equally the responsibilities which vest in the great array of professions which provide the intellectual and cultural strength of our society.

Fourth and finally, if American science is to continue to prosper, if it is to attract to it its proper complement of creative and gifted minds, we must combat the notions that science and engineering are incompatible with the great humanities, and that they are narrowly materialistic and destructive of human values. In the face of the practical responsibilities which rest in science and engineering for our security and our material welfare, it is all too easy for people to conclude that science is inimical to the spiritual ends of life and for them to fail to understand that in reality it is one of man's most powerful and noble means for searcing out truth and for augmenting man's dignity by augmenting his understanding. Scientists have an obligation to make this true character of science better understood, not by an arrogant advocacy of science and technology as the only objective means to increase our understanding and well-being, but by the balanced and tolerant practice and presentation of science as one of the powerful means by which man can increase his knowledge and understanding and still remain humble and ennobled before the wonder and the majesty of what he does not understand. When thus perceived and practiced, and when not misused for ignoble ends, science and engineering are major means for "making gentle the life of mankind." When so practiced and used, they become one of the great humanistic forces of our time.

THE INFLUENCE OF DEMAGNETIZATION AND ANISOTROPY ENERGY ON BLOCH WALL THICKNESS AND COERCIVE FORCE IN THIN FILMS

BY

ROBERT E. BEHRINGER [1] AND ROBERT S. SMITH [2]

ABSTRACT

A reasonable approximation is made to the actual Bloch wall structure in thin uniaxial magnetic films. The corresponding demagnetization, exchange, and anisotropy energies are calculated and the Bloch wall thickness is determined by minimizing the wall energy. The result is used to determine the functional relation between coercive force and film thickness on the surface roughness model of Néel. Results show that the inclusion of the demagnetization energy by the crude model of Néel is, in fact, a very good approximation. The inclusion of negative effective anisotropy leads to deviations from the four-thirds power law.

INTRODUCTION

The Bloch wall is the boundary region between magnetic domains. Within this region the magnetization, M, rotates from a direction parallel to M' in one domain to the direction parallel to M'' in the adjacent domain. When M' is antiparallel to M'', the wall is referred to as a 180° wall. The energy of the Bloch wall, γ, in bulk media, can be expressed as the sum of two terms,

$$\gamma = E_A + E_x,$$

where E_x is an exchange energy term, and E_A represents the energy due to the inclination of M with respect to some axis of anisotropy. The anisotropy axis is generally determined by a direction of strain or crystal orientation. The direction of M can be expressed as a function, $\theta(x)$, where x is the distance from a point in the wall to the center of the wall. θ is the angle which M makes with respect to any plane normal to the wall. E_A is given by $Kf(\theta)$ where $f(\theta)$ is some function of θ determined by the anisotropy of the material and E_x is $A\dfrac{d\theta^2}{dx}$. $\theta(x)$ will be that function which will minimize γ, so that Euler's equation becomes

$$Kf'(\theta) - 2A\frac{d\theta'}{dx} = 0. \tag{1}$$

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[2] General Products Division Development Laboratory, International Business Machines Corporation, San Jose, Calif.

This leads to the result

$$dx = \sqrt{\frac{A}{K}} \frac{d\theta}{[f(\theta)]^{1/2}}. \tag{2}$$

Here we have taken the yz-plane as the plane of the wall. From Eq. 2 we see that, knowing $f(\theta)$, we can easily calculate θ as a function of x and hence, the wall thickness.

Néel (1)[3] pointed out that in the case of thin films Eq. 2 is no longer applicable since there exists an appreciable demagnetizing energy. σ_D is generally minimized when M is parallel to the wall boundary so that free magnetic charges exist only on the top and bottom boundary of the film at the Bloch wall. To take account of the σ_D, Néel proposed that the wall be treated as an infinite ellipsoid with major axis of length $2D$ equal to the film thickness and minor axis of length $2a$ equal to the wall thickness magnetized along the major axis. The resulting demagnetizing energy per unit area of wall is readily seen to be

$$\sigma_D = \frac{\pi^2 a^2 M^2}{a + D}. \tag{3}$$

For thick films, that is, $D \gg a$, Eq. 3 becomes

$$\sigma_D = \frac{\pi^2 M^2 a^2}{D}. \tag{4}$$

Néel examined the film thickness dependence of the Bloch wall including σ_D and found that in some cases the energy σ is much more important than the anisotropy energy in determining wall thickness. Néel (2) assumed that as the wall moves under the influence of an applied field, γ will vary due to fluctuations in D. H_c would be that value of applied field for which the wall barely continues to move past the points of maximum change in γ. That is,

$$2MDH_c = \frac{2\gamma}{2D}\left(\frac{2D}{2x}\right)_{\max}.$$

He found for the case $K = 0$ that the coercive force should vary as the film thickness to the minus four-thirds power. Since Néel's model of the demagnetizing energy is a very crude approximation, it was felt that a more realistic model should be investigated to determine whether the four-thirds law is really significant. Furthermore, it was deemed worth-while to investigate the effect of anisotropy on this model, since in Fe–Ni films, the anisotropy is close to zero only near 80% Ni, 20% Fe.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

CALCULATION

Let us choose the plane of the film as the xy-plane and the plane of the Bloch wall as the yz-plane. If we assume the z-component of magnetization has the form $Mf(x)$ where $f(x)$ is a function of x only, then the induced surface charge density on the upper and lower surface of the film is $\pm Mf(x)$. The z-field component at the point $(u, 0, z)$ due to the charge $-Mf(x)$ on the lower surface is

$$H_z(u, z) = -\int_{-a}^{a} dx \int_{-\infty}^{\infty} dy \frac{zMf(x)}{[(x-u)^2 + y^2 + z^2]^{3/2}} \tag{5}$$

where $2a$ is the Bloch wall thickness and the film is assumed infinite in the y-direction. The energy per unit area required to separate the plus and minus charged sheets by the film thickness $2D$ is then

$$w_D(u) = \int_0^{2D} dz Mf(u) \int_{-a}^{a} dx \int_{-\infty}^{\infty} dy \frac{zMf(x)}{[(x-u)^2 + y^2 + z^2]^{3/2}} \tag{6}$$

and the demagnetizing energy per unit length of wall is

$$E_D = \int_{-a}^{a} du \int_0^{2D} dz Mf(u) \int_{-a}^{a} dx \int_{-\infty}^{\infty} dy \frac{zMf(x)}{[(x-u)^2 + y^2 + z^2]^{3/2}}. \tag{7}$$

If we carry out the z- and y-integrations, Eq. 7 reduces to

$$E_D = M^2 \int_{-a}^{a} du f(u) \int_{-a}^{a} dx f(x) \ln\left[1 + \frac{4D^2}{(x-u)^2}\right]. \tag{8}$$

Therefore, given the form of $f(x)$ we can in principle calculate the demagnetizing energy.

The function $f(x)$ is not completely arbitrary but must satisfy certain conditions. $f(x)$ must be symmetric about $x = 0$, that is, $f(-x) = f(x)$, $f(0)$ must equal unity, and near $x = \pm a$, $f(x)$ and $\frac{d\theta}{dx}$ must approach zero. We shall assume the conditions $f(\pm a) = 0$ and $\left.\frac{d\theta}{dx}\right|_{\pm a} = 0$ apply. The simplest function which satisfies these conditions is $\left(\frac{1 - x^2}{a^2}\right)^2$. However, this function is broader than the function obtained by Stephani (3) from a variational calculation for a material with cubic anisotropy. Since we would expect the function to be narrower for uniaxial material, we shall assume

$$f(x) = \left(1 - \frac{x^2}{a^2}\right)^3 \tag{9}$$

which is slightly narrower than the Stephani function.

If we replace u/a by u, x/a by x, and D/a by t and then substitute Eq. 9 in Eq. 8, we obtain

$$E_D = M^2a^2 \int_{-1}^{1} du(1-u^2)^3 \int_{-1}^{1} dx(1-x^2)^3 \ln\left[1 + \frac{4t^2}{(x-u)^2}\right]. \quad (10)$$

Now the integrand is symmetric under inversion and about $x = u$; therefore, we may write

$$E_D = 4M^2a^2 \int_{0}^{1} du(1-u^2)^3 \int_{-u}^{u} dx(1-x^2)^3 \ln\left[1 + \frac{4t^2}{(x-u)^2}\right]. \quad (11)$$

Equation 11 can be integrated and E_D determined exactly, but the result is in a form which is difficult to work with. Furthermore, we are interested in the case of thick films, that is, for films with $t \gg 1$. For these films the logarithmic term in the integrand simplifies to $2 \ln \left|\frac{2t}{x-u}\right|$ and we find

$$E_D = 1.673M^2a^2 (\ln t + 2.909). \quad (12)$$

For uniaxial material the anisotropy energy per unit length of wall is given by

$$E_K = 4DK \int_0^a \sin^2\theta dx$$

$$E_K = 4KaD \int_0^1 (1-x^2)^6 dx$$

$$E_K = 1.390KaD. \quad (13)$$

The corresponding exchange energy is

$$E_x = 4DA \int_0^a \left[\left(\frac{d\alpha_2}{dx}\right)^2 + \left(\frac{d\alpha_3}{dx}\right)^2\right] dx$$

$$E_x = \frac{144AD}{a} \int_0^1 dx \frac{x^2(1-x^2)^4}{[1-(1-x^2)^6]}$$

$$E_x = 12.66 \frac{AD}{a}. \quad (14)$$

The total energy per unit length of wall E can now be written as the sum of Eqs. 12, 13 and 14; thus

$$E = 1.673M^2a^2 \left(\ln \frac{D}{a} + 2.909\right) + 1.390KaD + 12.66 \frac{AD}{a}. \quad (15)$$

We note that the form of $f(x)$ does not affect the form of Eq. 15, but rather affects only the numerical coefficients. To determine the equilibrium wall thickness 2Δ, we minimize E with respect to a, that is,

$$\left.\frac{\partial E}{\partial a}\right|_{a=\Delta} = 0 = 3.346M^2\Delta\left(\ln\frac{D}{\Delta} + 2.409\right) + 1.390KD - 12.66\frac{AD}{\Delta^2}. \quad (16)$$

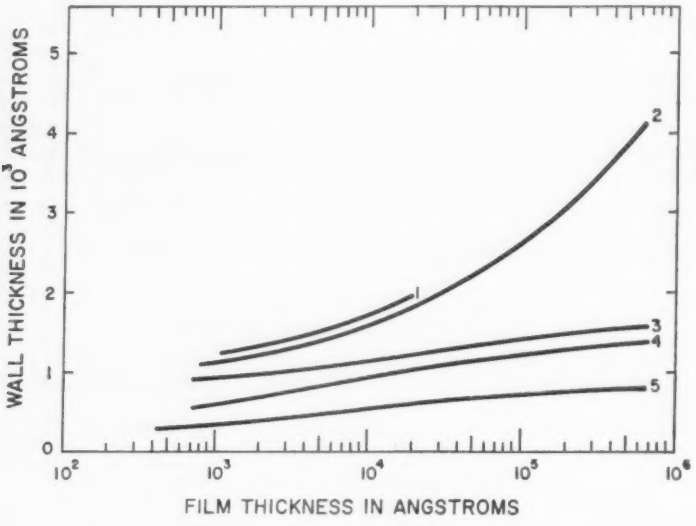

FIG. 1. Wall thickness *versus* film thickness—Néel model.

Curve	K, ergs/cc.	A, ergs/cm.	M, gauss
1	-2×10^5	3×10^{-6}	600
2	0	3×10^{-6}	600
3	2×10^5	3×10^{-6}	600
4	2×10^5	3×10^{-6}	1000
5	2×10^5	1×10^{-6}	1000

Solution of Eq. 16 gives Δ as a function of film thickness $2D$. Substituting this result into Eq. 15 then gives E as a function of film thickness. This result may then be used to calculate the coercive force H_c on the Néel (2) model according to which

$$H_c = (2MD)^{-1}\frac{\partial E}{\partial D}\left(\frac{\partial D}{\partial x}\right)_{max}. \quad (17)$$

If we set $\left(\frac{\partial D}{\partial x}\right)_{max}$ equal to S and calculate $\frac{\partial E}{\partial D}$ from Eq. 15, we

obtain

$$H_c = \frac{S}{2MD}\left(1.673\,\frac{M^2\Delta^2}{D} + 1.390K\Delta + 12.66\,\frac{A}{\Delta}\right) \tag{18}$$

where Δ is given by Eq. 16.

On the basis of the Néel (1) theory the equations corresponding to Eqs. 16 and 18 are

$$\left.\frac{\partial E}{\partial a}\right|_{a=\Delta} = 0 = \frac{2\pi^2M^2\Delta D(2D+\Delta)}{(\Delta+D)^2} + \pi KD - \frac{AD}{\Delta^2} \tag{19}$$

and

$$H_c = \frac{S}{2MD}\left[\frac{2\pi^2A}{\Delta} - \frac{4\pi^2M^2\Delta^2D}{(\Delta+D)^2}\right]. \tag{20}$$

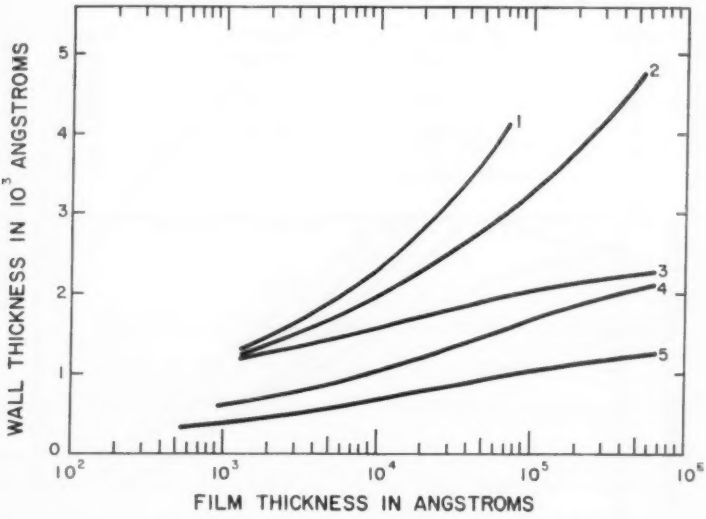

FIG. 2. Wall thickness *versus* film thickness—Behringer-Smith model.

Curve	K, ergs/cc.	A, ergs/cm.	M, gauss
1	-2×10^5	3×10^{-6}	600
2	0	3×10^{-6}	600
3	2×10^5	3×10^{-6}	600
4	2×10^5	3×10^{-6}	1000
5	2×10^5	1×10^{-6}	1000

DISCUSSION OF RESULTS

From Eqs. 16, 18, 19 and 20 it is clear that if K, A, and M are known, both Δ and H_c can be calculated as a function of D. In the materials

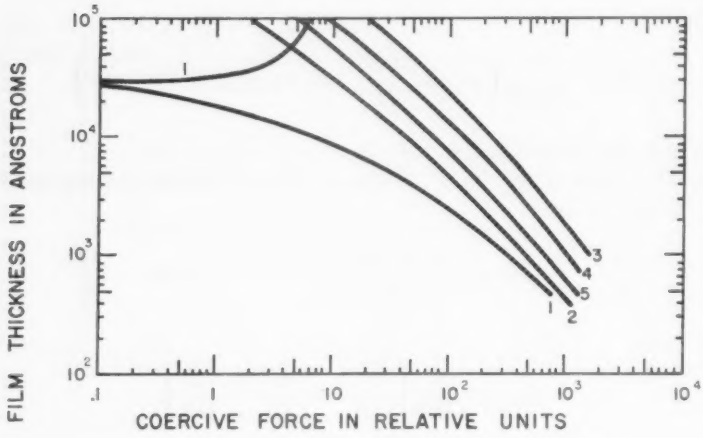

FIG. 3. Coercive force *versus* film thickness—Néel model.

Curve	*K*, ergs/cc.	*A*, ergs/cm.	*M*, gauss
1	-2×10^5	1×10^{-6}	1000
2	0	1×10^{-6}	1000
3	2×10^5	3×10^{-6}	600
4	2×10^5	3×10^{-6}	1000
5	2×10^5	1×10^{-6}	1000

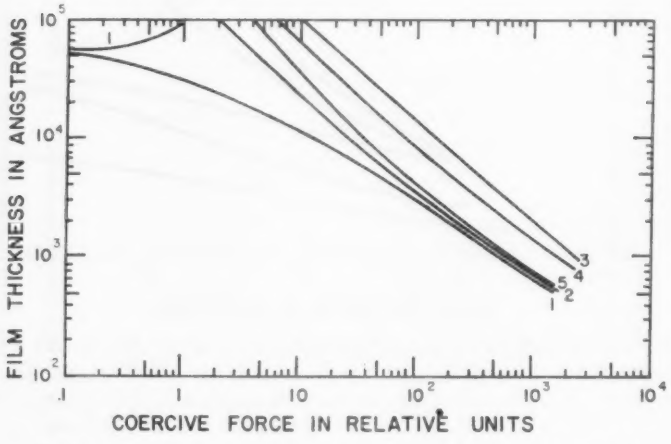

FIG. 4. Coercive force *versus* film thickness—Behringer-Smith model.

Curve	*K*, ergs/cc.	*A*, ergs/cm.	*M*, gauss
1	-2×10^5	1×10^{-6}	1000
2	0	1×10^{-6}	1000
3	2×10^5	3×10^{-6}	600
4	2×10^5	3×10^{-6}	1000
5	2×10^5	1×10^{-6}	1000

of interest, A ranges from 1 to 3 $\times$ 10^{-6} ergs/cm and M ranges from 600 to 1000 gauss. On the other hand, K can vary over an appreciable range depending on a host of deposition parameters. In Figs. 1 and 2, wall thickness is plotted as a function of film thickness on the Néel model and the present model, respectively. Plots of coercive force as a function of film thickness are given in Figs. 3 and 4. It is to be noted that the results of the Néel model agree qualitatively with the results of the present model. As indicated in the discussion following Eq. 15, the only effect is to shift the curves, the shapes remaining unaffected.

For materials with $K \geqslant 0$ the coercive force follows the four-thirds power law quite closely in the Bloch wall region. An interesting phe-

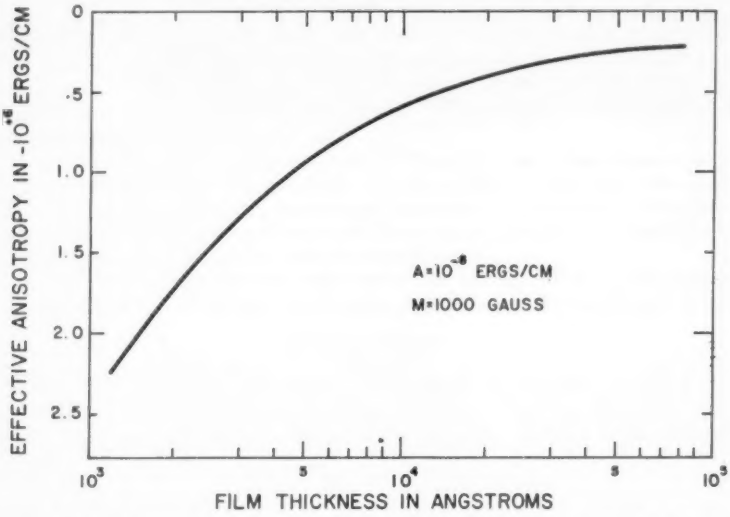

FIG. 5. Film thickness for $H_c = 0$ *versus* effective anisotropy—Behringer-Smith model.

nomenon occurs when the effective anisotropy is negative. In this case the coercive force vanishes at a film thickness determined by K, A, and M. For such materials there may or may not be a strong deviation from the four-thirds power law in the thin film region, depending upon the values of K, A, and M. Given A and M, it is possible to determine the film thickness for which the coercive force vanishes as a function of the effective anisotropy. The result for $A = 1 \times 10^{-6}$ ergs/cm and $M = 1000$ gauss is shown in Fig. 5.

We should like to emphasize at this point that our discussion of the coercive force is based entirely on the assumption that surface roughness is the source of coercive force. In any actual film this may not be true, as there are many other possible sources of coercivity. However, most other mechanisms are characteristic of the bulk material and

insensitive to film thickness. Those which are not characteristic of the bulk material, such as grain-size effects, would not result in a dependence of H_c on effective anisotropy as does the surface roughness model.

There is some evidence (4) that the four-thirds power law is followed for thin films with positive, effective anisotropy. For films with negative effective anisotropy, recent measurements (5) indicate a strong deviation from the four-thirds power law as predicted by the present calculation. It would thus appear that surface roughness does play an important role in determining the coercive force of thin metallic ferromagnetic films.

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ON INVERSES AND QUASI-INVERSES OF LINEAR TIME-VARYING DISCRETE SYSTEMS *

BY

ALAN B. MARCOVITZ [1]

ABSTRACT

This paper presents a solution to the general problem of finding inverses and quasi-inverses of time-varying linear discrete systems. The results are applicable to systems defined over any abstract field.

Unlike fixed systems, a time-varying system does not necessarily possess an inverse, even if a delay in the output of the inverse is allowed. This inverse with delay is called a quasi-inverse.

The necessary and sufficient conditions for the existence of a quasi-inverse are established. These conditions are difficult to apply, since they often require an infinite number of calculations. However, a simple necessary condition and a simple sufficient condition are derived.

The existence of a quasi-inverse does not insure that the inverse system can be represented by a unique canonical form or a unique difference equation. Necessary and sufficient conditions for the existence of a unique inverse are shown. Under these same conditions, the canonical form of the inverse is the same as the canonical form of the inverse of a fixed system with the same structure.

The necessary and sufficient conditions for the special case of a coding system where the system is described by a transmission matrix of finite dimension are given.

I. INTRODUCTION

It is the purpose of this paper to present a solution to the problem of obtaining inverses or quasi-inverses of time-varying discrete systems (1, 2).[2] One method of description of a discrete system is the difference equation

$$y(n) + b_1(n)y(n-1) + \cdots + b_m(n)y(n-m) = a_0(n)x(n) + a_1(n)x(n-1) + \cdots + a_m(n)x(n-m). \quad (1)$$

It is assumed that the system is physical and thus $y(n)$ depends only on the current input and past values of inputs and outputs, not on future values of the input.

Another representation of a linear system that will be useful in the analysis of time-varying systems is that of matrix notation (4). The superposition summation

$$y(n) = \sum_{i=0}^{n} h_{n-i}(i)x(i) \quad (2)$$

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

relates the input to the output of any linear system.

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \end{bmatrix} = \begin{bmatrix} h_0(0) & 0 & 0 & 0 & \cdots \\ h_1(0) & h_0(1) & 0 & 0 & \cdots \\ h_2(0) & h_1(1) & h_0(2) & 0 & \cdots \\ \vdots & & & & \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \end{bmatrix} \tag{3}$$

The matrix of h's will be called the transmission matrix and will be denoted by **H**. A necessary and sufficient condition for **H** to represent a physically realizable system is that **H** is lower triangular as indicated in (3). This corresponds to taking the upper limit in (2) at n rather than ∞. It can readily be verified by examining (3) that the i^{th} column of **H** is the response of the system to a unit input at time $i - 1$. Note also that $h_0(n)$ is identical to $a_0(n)$ in the difference equation formulation. The unit response of a discrete system is defined as the response of the system to an input of the form

$$x(n) = \begin{cases} 1 & n = i \\ 0 & n \neq i \end{cases} \tag{4}$$

and is

$$y(n) = \begin{cases} h_{n-i}(i) & n \geq i \\ 0 & n < i \end{cases} \tag{5}$$

Two separate cases will be considered. In the first, the unit response of the system is defined for all $n \geq 0$ and is not necessarily identically zero for all $n > n_0$ for any $n_0 \geq 0$. For this case, the quasi-inverse of a system H whose output is $y(n)$ when its input is $x(n)$ is defined as that system H' whose output is $x(n - k)$ when its input is $y(n)$ where $0 \leq k < \infty$.

The other type of system is one for which the input consists of a finite number of information digits followed by a fixed (finite) number of zeroes and the output consists of a finite number of digits (equal to the total number of input digits, including zeroes). This type of system has applications in sequential coding networks. Given a system of this type H with an output $y(n)$ for an input $x(n)$, where $x(n)$ contains m information digits followed by r zeroes, the quasi-inverse H' is that system which produces an output $x'(n - k)$, where $x'(n - k)$ consists of k zeroes followed by the m information digits of $x(n)$ followed by $r - k$ zeroes. In both cases when k (the delay) is zero, H' is a true inverse, referred to simply as the inverse and denoted by H^{-1}. If k is greater than zero, the quasi-inverse is referred to as an inverse with time delay. The minimum delay k_m is defined as the smallest value of k for which a quasi-inverse exists.

The results presented in this paper are applicable to systems where

the inputs are over finite fields, such as in linear modular sequential circuits (5, 6), as well as the complex number fields as in digital control systems.

The first problem considered is that of obtaining the necessary and sufficient conditions for the existence of a quasi-inverse. Once these are found the second problem is to determine whether a system having a quasi-inverse can be represented by a unique difference equation. Finally, it is necessary to find a method of obtaining the difference equation or some other representation of the quasi-inverse system.

This problem differs from the general problem of inverses of fixed systems in that a time-varying system does not always possess a quasi-inverse. Furthermore, even if a quasi-inverse does exist, its form and the coefficients in its difference equation are not necessarily simply related to the difference equation of the original system. Although the input-output relation can be represented by (3), the problem differs from the general problem of finding the inverse of a matrix since the quasi-inverse is defined by the equation

$$\mathbf{H'H} = \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{I} & \mathbf{O} \end{bmatrix} \tag{6}$$

and is not equal to the product in the reverse order $\mathbf{HH'}$ except in the special case where $k = 0$. Furthermore, $\mathbf{H}$ may be a semi-infinite matrix.

The conditions for the existence of the quasi-inverse of a fixed system are well known. The quasi-inverse always exists. If, in addition, $a_0(n) \neq 0$ which is equivalent to $h_0(n) \neq 0$, a true inverse (zero delay) exists. The latter condition, $h_0(n) \neq 0$ is precisely the necessary and sufficient condition for the existence of an inverse of a triangular matrix. For a fixed system, the coefficients of the difference equation and the terms of the transmission matrix are not functions of n and are denoted by a_i, b_i and h_i, respectively.

For a fixed system, where the initial conditions are assumed zero, the system can be represented by a transfer function $H(z)$, where

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-k}(a_k + a_{k+1}z^{-1} + \cdots + a_m z^{-(m-k)}}{1 + b_1 z^{-1} + \cdots + b_m z^{-m}} \tag{7}$$

where $a_i = 0$ for $i < k$ and $X(z)$ and $Y(z)$ are the Z-transforms of the input and output, respectively. The transfer function can also be written in terms of the h_i as

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} + \cdots \tag{8}$$

where $h_i = 0$ for $i < k$ if $a_i = 0$ for $i < k$ and the summation may include an infinite number of terms.

The transfer function of the quasi-inverse system is the reciprocal of $\frac{H(z)}{z^{-k}}$ and is denoted by $H'(z)$.

$$H'(z) = \frac{1 + b_1 z^{-1} + \cdots + b_m z^{-m}}{a_k + a_{k+1} z^{-1} + \cdots + a_m z^{-(m-k)}} \tag{9}$$

$$= \frac{\frac{1}{a_k} + \frac{b_1}{a_k} z^{-1} + \cdots + \frac{b_m}{a_k} z^{-m}}{1 + \frac{a_{k+1}}{a_k} z^{-1} + \cdots + \frac{a_m}{a_k} z^{-(m-k)}} \tag{10}$$

$$= \frac{b_0' + b_1' z^{-1} + \cdots + b_m' z^{-m}}{1 + a_1' z^{-1} + \cdots + a'_{m-k} z^{-(m-k)} + \cdots + a_m' z^{-m}} \tag{11}$$

where $a_i' = 0$ for $m - k < i \leq m$. Figure 1 shows one possible circuit for the realization of the system and Fig. 2 shows a possible circuit for the realization of the quasi-inverse. Note that the quasi-inverse is of the same form and structure as the original circuit, with the only difference being the values of the gains. The gains are directly determined from the original circuit, as can be seen from (10) and (11).

The quasi-inverse can also be obtained from the transmission matrix. Consider first an infinite transmission matrix of the form

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots & \\ 0 & \vdots & \vdots & \\ h_k & 0 & \vdots & \\ h_{k+1} & h_k & 0 & \\ h_{k+2} & h_{k+1} & h_k & 0 \cdots 0 \end{bmatrix} = \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{K} & \mathbf{O} \end{bmatrix} \tag{12}$$

where K is a square[3] lower triangular matrix, with all non-zero terms on the major diagonal. Therefore $\mathbf{K}$ possesses an inverse $\mathbf{K}^{-1}$. The system described by $\mathbf{K}^{-1}$ is the quasi-inverse of $\mathbf{H}$. To prove this, consider the outputs of systems described by $\mathbf{K}$ and $\mathbf{H}$.

$$\mathbf{Y}_1 = \mathbf{KX} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \end{bmatrix} \tag{13}$$

[3] Although $\mathbf{K}$ and $\mathbf{H}$ are infinite, it is convenient to think of them as square. Thus an inverse (also infinite) may be defined in a similar manner to that for finite matrices.

$$\mathbf{Y}_2 = \mathbf{HX} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ y(0) \\ y(1) \\ \vdots \end{bmatrix} \begin{matrix} \overline{\uparrow} \\ k \\ \underline{\downarrow} \end{matrix} = \begin{bmatrix} \mathbf{O} \\ \mathbf{Y}_1 \end{bmatrix}. \tag{14}$$

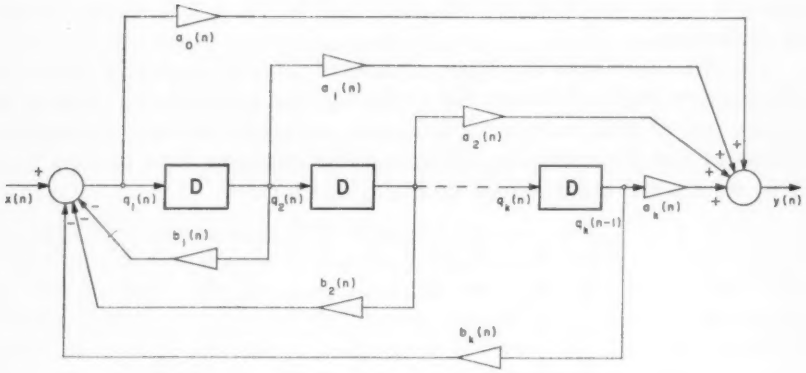

FIG. 1. The circuit H.

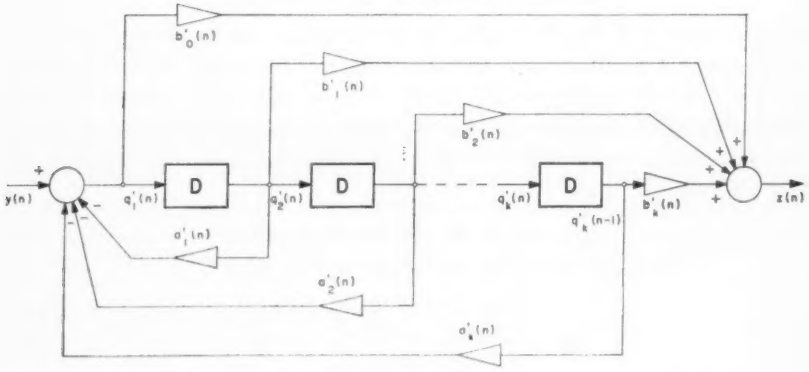

FIG. 2. The inverse circuit H^{-1}.

If each of these is now passed through K^{-1}, the outputs are

$$\mathbf{K}^{-1}\mathbf{Y}_1 = \mathbf{K}^{-1}\mathbf{KX} = \mathbf{X} \tag{15}$$

$$\mathbf{K}^{-1}\mathbf{Y}_2 = \begin{bmatrix} \mathbf{O} \\ \mathbf{X} \end{bmatrix}. \tag{16}$$

The latter is obvious if one considers that **H** is infinite and

$$\mathbf{K} = \begin{bmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{B} & \mathbf{K} \end{bmatrix} \tag{17}$$

where **A** is any square submatrix and could be the $k \times k$ square matrix in particular.

For the case where the input consists of a finite number of digits, m information digits followed by r zeroes, the transmission matrix is square and of order $m + r$. The matrix must still be lower triangular. If the output also contains a finite number of digits, $h_i = 0$, for $i > r$. This results in a transmission matrix of the form

$$\mathbf{H} = \begin{bmatrix} h_0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ h_1 & h_0 & 0 & 0 & \cdots & 0 & \\ \multicolumn{7}{c}{\text{- -}} \\ h_r & h_{r-1} & \cdots & h_0 & 0 & \cdots 0 & 0 \\ 0 & h_r & \cdots & h_1 & h_0 & 0 \cdots 0 & 0 \\ \multicolumn{7}{c}{\text{- -}} \\ 0 & 0 \cdots h_r & h_{r-1} & \cdots & & h_0 & 0 \\ 0 & \cdots 0 & h_r & h_{r-1} & \cdots & h_1 & h_0 \end{bmatrix}. \tag{18}$$

As an example, consider the coding network represented by the finite matrix with $m = 4$ and $r = 3$. All operations in the system are modulo two.

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \tag{19}$$

In this case, a true inverse exists, since all the terms on the major diagonal are nonzero. For the example of (19) the inverse is

$$\mathbf{H}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}. \tag{20}$$

The transfer functions corresponding to these matrices are

$$H(z) = 1 + z^{-1} + z^{-3} \tag{21}$$

$$H^{-1}(z) = \frac{1}{1 + z^{-1} + z^{-3}}. \tag{22}$$

From (22) it can be seen that there are non-zero values for h_i for $i > r$. Since the inverse system is also fixed, this would imply that the output of the inverse must be considered for $n \geq m + r$. This would also imply that the matrix of the inverse system is of infinite dimension with (20) only a submatrix. However if $\mathbf{H}$ and $\mathbf{H}^{-1}$ are cascaded and it is assumed that $y(n)$ for $n \geq m + r$ is zero, it can be seen that the output of the inverse is also zero for $n \geq m + r$.

II. EXISTENCE AND UNIQUENESS THEOREMS

In this section the existence of a quasi-inverse of a time-varying system will be discussed. Two theorems—one for the infinite transmission matrix and one for the finite transmission matrix—will be proved. In addition, the uniqueness of the matrix or difference equation describing the quasi-inverse will be discussed.

We first consider infinite transmission matrices. In order to prove the theorem for the existence of a quasi-inverse of a time-varying system with an infinite transmission matrix, it is necessary to prove three lemmas. The proofs of these lemmas are given in Appendix I.

Lemma 1: Given a square matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{B} \\ \mathbf{I} & \mathbf{D} \end{bmatrix}$$

where $\mathbf{I}$ is the $n \times n$ identity matrix, and $\mathbf{D}$ is an arbitrary $r \times r$ matrix, the rank of $\mathbf{A}$, $R(\mathbf{A}) = R(\mathbf{I}) + R(\mathbf{B})$.

Lemma 2: Consider a set of r independent equations in $i + k > r$ unknowns. After completion of the Gauss-Jordan reduction, they

may be written as

$$\begin{aligned} x_1 &= c_1 + \alpha_{11}x_{r+1} + \cdots + \alpha_{1,i+k-r}x_{i+k} \\ x_2 &= c_2 + \alpha_{21}x_{r+1} + \cdots + \alpha_{2,i+k-r}x_{i+k} \\ &\text{- -} \\ x_i &= c_i + \alpha_{i1}x_{r+1} + \cdots + \alpha_{i,i+k-r}x_{i+k} \\ &\text{- -} \\ x_r &= c_r + \alpha_{r1}x_{r+1} + \cdots + \alpha_{r,i+k-r}x_{i+k} \end{aligned} \tag{23}$$

The necessary and sufficient condition for the existence of a unique solution for $x_1, x_2, \cdots, x_i$ is that $\alpha_{pq} = 0$ for $1 \leq p \leq i$ and $1 \leq q \leq i + k - r$.

Lemma 3: If a matrix $\mathbf{A}$ is partitioned into two submatrices such that $\mathbf{A} = [\mathbf{A}_1\ \mathbf{A}_2]$, then the rank of $\mathbf{A}$, $R(\mathbf{A}) \leq R(\mathbf{A}_1) + R(\mathbf{A}_2)$.

The notation to be used in the theorem and its proof is illustrated in (24) and is as follows: $\mathbf{H}_i$ is the $i \times i$ submatrix of $\mathbf{H}$ containing the first i rows and columns of $\mathbf{H}$; $\mathbf{H}_{ik}$ is the submatrix of $\mathbf{H}_{i+k}$ containing the first i columns of $\mathbf{H}_{i+k}$; and $\mathbf{H}_{ik}'$ is that submatrix of $\mathbf{H}_{i+k}$ containing the last k columns of $\mathbf{H}_{i+k}$.

$$\mathbf{H}_{i+k} = \left[\begin{array}{c:c} \overset{\leftarrow i \rightarrow}{\mathbf{H}_{ik}} & \overset{\leftarrow k \rightarrow}{\mathbf{H}_{ik}'} \end{array}\right] \begin{matrix} \uparrow \\ i + k \\ \downarrow \end{matrix} \tag{24}$$

The rank of $\mathbf{H}_{i+k}$ is r. The rank of $\mathbf{H}_{ik}'$ is r'.

Theorem 1: The necessary and sufficient conditions for the existence of a quasi-inverse with a delay of k units of time are that for all i

(1) The rank of $\mathbf{H}_{ik}$ is i.
(2) The rank of $\mathbf{H}_{ik}'$ is $r - i$.

Proof: (*a*) Sufficiency

If the rank of $\mathbf{H}_{ik}$ is i, it is possible by appropriate manipulation of the rows (multiplication by a constant and addition of two rows) to obtain a new $\mathbf{H}_{i+k}$ such that there is exactly one unit element in each column of $\mathbf{H}_{ik}$ and no more than one unit element in each row of $\mathbf{H}_{ik}$. All other terms of $\mathbf{H}_{ik}$ are zero. This can be accomplished by the Gauss-Jordan reduction on the set of equations represented by this matrix. By rearranging the rows of $\mathbf{H}_{i+k}$, it can be put in the form

$$\mathbf{H}_{i+k} = \begin{bmatrix} \mathbf{I} & \mathbf{E} \\ \mathbf{O} & \mathbf{F} \end{bmatrix} \tag{25}$$

where $\mathbf{I}$ is the $i \times i$ identity matrix. None of the operations to this point has changed the rank of $\mathbf{H}_{i+k}$, $\mathbf{H}_{ik}$ or $\mathbf{H}_{ik}'$. These operations have not affected the solution of the associated equations. Since the rank of $\mathbf{H}_{i+k}$ is r, the Gauss-Jordan reduction can be continued, so that after rearranging rows within $[\mathbf{O}\ \mathbf{F}]$ and columns within $\mathbf{H}_{ik}'$, if necessary, $\mathbf{H}_{i+k}$ is of the form

$$\mathbf{H}_{i+k} = \begin{bmatrix} \mathbf{I}_1 & \mathbf{O} & \mathbf{K}_1 \\ \mathbf{O} & \mathbf{I}_2 & \mathbf{K}_2 \\ \mathbf{O} & \mathbf{O} & \mathbf{O} \end{bmatrix} \tag{26}$$

where $\mathbf{I}_1$ is the $i \times i$ identity matrix and $\mathbf{I}_2$ is the $(r - i) \times (r - i)$ identity matrix. If condition (2) holds, then r', which equals the sum of the ranks of $\mathbf{I}_2$ and $\mathbf{K}_1$ (by Lemma 1) must be less than or equal to $r - i$. But, the rank of $\mathbf{I}_2$ is equal to $r - i$ and therefore the rank of $\mathbf{K}_1$ must equal zero. If the rank of a matrix is zero, then all its elements must be zero. Returning to the equations which this matrix represents, they are in the form of Lemma 2, where

$$\mathbf{K}_1 = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1,i+k-r} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2,i+k-r} \\ \alpha_{i1} & \alpha_{i2} & \cdots & \alpha_{i,i+k-r} \end{bmatrix} \tag{27}$$

Since $\mathbf{K}_1 = \mathbf{O}$, the conditions of the lemma are met, and a unique solution exists for the first i variables, from the first $i + k$ equations. By induction, it is now possible to show that it is possible to obtain a quasi-inverse with delay k. Note that an inverse with delay k means that at time $i + k$, the output of the inverse circuit must be the input to the original circuit at time i. If the theorem is true for $i = 0$, then at time k it is possible to solve for $x(0)$ and the output of the inverse would be $x(0)$. If an inverse exists for $i = n$, then the output at time k is $x(0)$, at time $k + 1$ is $x(1)$, and so forth to the output at time $n + k$ is $x(n)$. By the theorem, it is possible to solve for the $(n + 1)^{\text{th}}$ input by time $n + k + 1$ and thus the output at time $n + k + 1$ would be $x(n + 1)$. Thus, if a quasi-inverse exists to time n, it exists to time $n + 1$ and sufficiency has been proved.

(*b*) Necessity

To prove necessity, it will be shown that if either of the conditions is violated, then a quasi-inverse with delay k does not exist. If condition (1) is violated, then the rank of $\mathbf{H}_{ik}$ is $j < i$. By following the same procedure as before, it is possible to reduce $\mathbf{H}_{i+k}$ (by rearranging columns within $\mathbf{H}_{ik}'$ if necessary) to the form

$$\mathbf{H}_{i+k} = \left[\begin{array}{c|c|c} \mathbf{I} & \mathbf{J} & \\ \hline & & \mathbf{H}_{ik}' \\ \hline \multicolumn{2}{c|}{\mathbf{O}} & \end{array}\right] \tag{28}$$

where $\mathbf{I}$ is a $j \times j$ unit matrix and $\mathbf{H}_{ik}'$ has been modified by the operations on the rows. The j equations represented by the first j rows of this modified matrix are the only equations involving the first i unknowns. Since there are j equations in $i > j$ unknowns, a unique solution does not exist for the first i unknowns and thus a quasi-inverse with delay k does not exist. It is impossible for r' to be less than $r - i$ by Lemma 3. Therefore, if condition (2) is to be violated, r' must be greater than $r - i$. We can assume that the rank of $\mathbf{H}_{ik}$ equals i, since we have shown that if this were violated, a quasi-inverse cannot exist. Since the rank of $\mathbf{H}_{i+k}$ is r and that of $\mathbf{H}_{ik}$ is i, $\mathbf{H}_{i+k}$ can be reduced to the form of (26) as before. By Lemma 1, the rank of $\mathbf{H}_{ik}'$, $r' = R(\mathbf{I}_2) + R(\mathbf{K}_1)$, which equals $r - i + R(\mathbf{K}_1)$. Since $r' > r - i$, $R(\mathbf{K}_1)$ must be greater than zero and $\mathbf{K}_1$ cannot be identically zero. Since $\mathbf{K}_1$ is not identically zero, the α_{pq} $(1 \leq p \leq i)$ of the equations represented by this matrix are not all zero and thus by Lemma 2, a unique solution for $x_1, x_2, \cdots, x_i$ does not exist. Thus a quasi-inverse with delay k does not exist if either of the conditions is violated, and, therefore, the theorem is proved.

This theorem, although mathematically simple, is quite difficult to apply in practice since an infinite number of calculations are generally required. If, however, a particular situation requires a quasi-inverse for only a finite time b, then the matrix must satisfy the theorem only for $0 \leq i \leq b$.

In addition to the necessary and sufficient conditions presented in Theorem 1, two other partial criteria exist which may be very useful.

Corollary 1: The sufficient conditions for the existence of a quasi-inverse with minimum delay k are that

$$\left.\begin{array}{lll} (1) & h_i(n) = 0 & i < k \\ (2) & h_k(n) \neq 0 & \end{array}\right\} \text{for all } n$$

Proof: Consider the form of the transmission matrix for this special case. For an arbitrary i,

$$\mathbf{H}_{i+k} = \begin{array}{c} \uparrow \\ k \\ \downarrow \end{array} \begin{bmatrix} 0 & 0 \cdots & & & \\ \vdots & \vdots & & & \\ 0 & \vdots & & & \\ h_k(0) & 0 & & & \\ h_{k+1}(0) & h_k(1) & 0 \cdots & & \\ h_{k+2}(0) & h_{k+1}(1) & h_k(2) & 0 \cdots & \\ h_{i+k-1}(0) & h_{i+k-2}(1) & \cdots & h_k(i-1) & 0 \cdots 0 \\ & & & & \leftarrow k \rightarrow \end{bmatrix} \tag{29}$$

It is clear from an examination of (29) that if $h_k(n) \neq 0$ for all n, the determinant of the $i \times i$ submatrix consisting of the first i columns and last i rows of $\mathbf{H}_{i+k}$ is non-zero and thus the rank of $\mathbf{H}_{i+k}$ is i. This submatrix falls entirely within $\mathbf{H}_{ik}$ and thus the rank of $\mathbf{H}_{ik}$ is also i, which satisfies condition (1) of Theorem 1. Finally, $\mathbf{H}_{ik}' = 0$, and thus is of rank zero, so that $r' = 0 = r - i = i - i$ and the theorem is satisfied.

Corollary 2: If $h_i(n) = 0$ for $i < q$ and for all n, and $h_q(n) \neq 0$ for some n, a necessary condition for the existence of a quasi-inverse is that $h_q(n) = 0$ either for an infinite number of n or for no n.

Proof: To prove this theorem, let us assume that $h_q(n) = 0$ for only a finite number of n. Therefore, since the transmission matrix is of infinite size, there must exist an i such that $h_q(i-1) = 0$ and $h_q(j) \neq 0$ for all $j \geq i$. This results from the fact that there must be a largest number in a finite set of numbers. For this i, consider the matrix $\mathbf{H}_{i+k}$.

$$\mathbf{H}_{i+k} = \begin{array}{c} \leftarrow i \rightarrow \end{array} \begin{array}{c} \leftarrow k \rightarrow \end{array} \begin{bmatrix} 0 & 0 & & \\ 0 & \vdots & & \\ \vdots & \vdots & & \\ 0 & \vdots & & \\ h_q(0) & 0 & & \\ h_{q+1}(0) & h_q(1) & & \\ & & \ddots & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & h_q(i+k-q-1) & 0 \cdots 0 \\ & & & \leftarrow q \rightarrow \end{bmatrix} \tag{30}$$

For any k, the rank of $\mathbf{H}_{i+k}$, called r, must be less than $i + k - q$, since some $h_q(n) = 0$ for $n < i$. The rank of $\mathbf{H}_{ik}'$ must equal $k - q$ if $k - q > 0$ or zero otherwise, since $h_q(j) \neq 0$ for $j \geq i$. The rank of $\mathbf{H}_{ik}$ must be less than or equal to i, its smaller dimension. If the rank is less than i, an inverse does not exist by condition (1) of Theorem 1. If the rank equals i, condition (2) states that $r - i = r'$. But

$$r - i < i + k - q - i = k - q = r'. \tag{31}$$

Thus

$$r' > r - i \tag{32}$$

and condition (2) of the theorem is violated. Therefore, a quasi-inverse cannot exist and the corollary is proved.

This corollary is clear if the equations involved are considered. Since $h_q(i - 1) = 0$, the first q equations involve at most $i - 1$ unknowns. Since $h_q(i) \neq 0$, the $(q + 1)^{\text{th}}$ equation introduces $x(i + 1)$ and possibly $x(i)$. Each additional equation introduces at least one new unknown. It is thus impossible to solve for $x(i)$ at any time.

A system possesses a unique quasi-inverse if and only if a quasi-inverse exists and all the terms of the transmission matrix of the quasi-inverse are uniquely determined. This definition does not imply that the realization is unique, since any system, fixed or time-varying, can be realized in many ways. It does imply that the difference equation of the quasi-inverse system is unique except for $a_i'(j)$ and $b_i'(j)$ for $j < i$. (The exception is implied by the fact that the multipliers of these coefficients $x(j - i)$ and $y(j - i)$ are zero for $j < i$.) Thus a system whose quasi-inverse is unique possesses a unique canonical form as shown in Fig. 2.

Theorem 2: The necessary and sufficient conditions for the existence of a unique quasi-inverse are that

$$\left.\begin{aligned} h_i(n) &= 0 \qquad i < k \\ h_k(n) &\neq 0 \end{aligned}\right\} \text{for all } n$$

Proof: (*a*) Sufficiency: If the conditions of the theorem are met, then a lower triangular matrix can be obtained by deleting the first k rows of $\mathbf{H}$. This matrix has the property that all elements on the major diagonal are nonzero. The inverse (by mathematical definition) of this matrix exists using standard inversion techniques and is lower triangular. Thus a unique inverse exists.

(*b*) Necessity: Assume that the conditions are not met, that is, $h_k(n) = 0$ for some (but not all) n. Choose the smallest n for which $h_k(n) = 0$ and denote this by j. Consider the solution for the j^{th} row of the quasi-inverse of $\mathbf{K}$, that submatrix of $\mathbf{H}$ obtained by deleting the first k rows and the last k columns of $\mathbf{H}$. (See (12).) Since the deleted

rows and columns contain only zeroes, the quasi-inverse of $\mathbf{K}$ must also be the quasi-inverse of $\mathbf{H}$. The defining equation of $\mathbf{K}'$ is

$$\mathbf{K}'\mathbf{K} = \boldsymbol{\phi} \tag{33}$$

or

$$\begin{bmatrix} f_{00} & 0 & 0 & \cdots\cdots\cdots \\ f_{10} & f_{11} & 0 & \\ \text{---} & \text{---} & \text{---} & \text{---} \\ f_{j0} & f_{j1}\cdots f_{jj} & 0\cdots & \\ \text{---} & \text{---} & \text{---} & \text{---} \end{bmatrix} \begin{bmatrix} h_k(0) & 0 & 0 & 0\cdots \\ h_{k+1}(0) & h_k(1) & 0 & 0\cdots \\ h_{k+2}(0) & h_{k+1}(1) & h_k(2) & 0\cdots \\ \text{---} & \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} & \text{---} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0\cdots \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0\cdots \\ 1 & 0 & 0\cdots \\ 0 & 1 & 0\cdots \end{bmatrix}. \tag{34}$$

The matrix $\boldsymbol{\phi}$ cannot be the identity matrix since this would imply that $\mathbf{K}$ possesses an inverse which is clearly impossible since $\mathbf{K}$ is singular. In particular $\phi_{jj} = 0$. Consider now the set of equations involving the elements of row j of $\mathbf{K}'$.

$$\begin{aligned} h_k(0)f_{j0} + h_{k+1}(0)\quad f_{j1} + \cdots + h_{k+j}(0)\quad f_{jj} &= \phi_{j0} \\ h_k(1)\quad f_{j1} + \cdots + h_{k+j-1}(1)f_{jj} &= \phi_{j1} \\ \text{- - - - - - - - - - - - - - - - -} & \\ h_k(j)f_{jj} &= \phi_{jj} \end{aligned} \tag{35}$$

It was assumed that $h_k(j) = 0$, and $h_k(n) \neq 0$ for $n < j$. Thus, there are j equations in $j + 1$ unknowns unless $h_{k+j}(0) = h_{k+j-1}(1) = \cdots = h_{k+1}(j-1) = 0$. In the former case, a unique solution does not exist since there are more equations than unknowns. In the latter case, a unique solution does not exist since f_{jj} is undetermined. Therefore a unique quasi-inverse does not exist and the theorem is proved.

Systems with finite $\mathbf{H}$ matrices are encountered in coding circuits. After being coded, the information (including checking digits) is passed through a noisy channel and decoded by the quasi-inverse of the coding system. Thus, the quasi-inverse of a system with a finite matrix is desired.

If the input consists of r information digits followed by k zeroes, it is obvious that the last k columns of the coding matrix are arbitrary, in that their value has no effect on the output. Denoting that submatrix of $\mathbf{H}$ which is composed of the first r columns of $\mathbf{H}$ as $\mathbf{H}'$, the following existence theorem can be stated.

Theorem 3: The necessary and sufficient condition for the existence of a quasi-inverse of a finite matrix is that the rank of $\mathbf{H}'$ is r. If $\mathbf{H}_r'$ is defined as the $r \times r$ submatrix containing the first r independent rows of $\mathbf{H}'$, then the maximum delay is the number of the last row of $\mathbf{H}_r'$ (where the first row of $\mathbf{H}'$ is numbered zero.)

Proof: The proof of this theorem can be divided into three parts—sufficiency, necessity and maximum delay.

(1) Sufficiency: If the theorem is satisfied, then $\mathbf{H}_r'$ is a matrix relating the r information digits to r of the outputs. Since this matrix is of rank r, it is possible to solve for the r information digits in terms of the r outputs. Thus, a quasi-inverse exists.

(2) Necessity: If the theorem is violated, then the rank of $\mathbf{H}'$ must be less than r (since the rank of a matrix must be less than or equal to its smallest dimension). But if the rank of $\mathbf{H}'$ is $q < r$ there are only q independent equations in r unknowns, and it is thus impossible to solve for the r information digits. Therefore, a quasi-inverse does not exist.

(3) Maximum Delay: Consider the $f \times r$ submatrices of $\mathbf{H}'$ which contain the first f rows of $\mathbf{H}'$. There is a smallest value of $f = f_m$ for which the submatrix described is of rank r. $\mathbf{H}_r'$ is the submatrix of this smallest submatrix which contains the r rows that make up the $r \times r$ non-zero determinant. Row f_m must be one of these (since the rank would be r for a smaller f otherwise). This is equivalent to stating that $y(0), \cdots, y(f)$ are sufficient to determine the input and therefore all the inputs are determined by time f. The output of the quasi-inverse can then be $x(0)$ at time f_m and thus a delay of f_m at most is required. A shorter delay is possible if $x(0)$ can be obtained before time f_m.

It can readily be seen that if $\mathbf{H}_r'$ is a lower triangular matrix (which implied that all elements of the major diagonal are non-zero from the definition of $\mathbf{H}_r'$) that the delay is equal to the $f_m - r + 1$, that is, the i^{th} input digit can be found at the time of the i^{th} independent row and thus the r^{th} input digit is found at time f_m. Further, it can be seen that $\mathbf{H}_r'$ possesses a unique inverse in this case. The most common special case used in coding networks is where $\mathbf{H}_r'$ is the first r rows of $\mathbf{H}$ and thus a unique inverse with no delay exists. The transmission matrix $\mathbf{H}$ would then be of the form

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_r' & \mathbf{O} \\ \mathbf{C} & \boldsymbol{\phi} \end{bmatrix}$$

where $\boldsymbol{\phi}$ is an optional matrix and $\mathbf{C}$ depends only upon the error correction scheme used. If $\mathbf{H}_r'$ is a unit matrix, which would be the case for a Hamming type coder (7) and $\boldsymbol{\phi}$ is chosen also as a unit matrix, the inverse matrix would be

$$\mathbf{H}' = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ -\mathbf{C} & \mathbf{I} \end{bmatrix}$$

and thus the inverse is structurally identical to the coder with some gains replaced by their additive inverse (7). In the case of a binary system $\mathbf{H}' = \mathbf{H}$.

III. A CANONICAL FORM OF THE QUASI-INVERSE

Although a simple general procedure for constructing the quasi-inverse is not available, a partial solution of this problem does exist. Firstly, in the special case where $h_i(n) = 0$ for $i < k$ and $h_k(n) \neq 0$ for all n, the canonical form of the quasi-inverse is of the form of the original network. This will be proved as a theorem below. Secondly, in all cases where quasi-inverses do exist, the matrix of the quasi-inverse can readily be obtained. Examples of both of these will be given below.

Theorem 4. For the circuit shown in Fig. 1 and described by (1), if the gains $a_i(n)$ and $b_i(n)$ are not constants but

$$\left.\begin{aligned} a_i(n) &= 0 \qquad i < k \\ a_k(n) &\neq 0 \end{aligned}\right\} \text{ for all } n$$

then the quasi-inverse of the time variant circuit exists and is of the form of Fig. 2. The values of the multipliers are given by

$$a_i'(n) = \begin{cases} \dfrac{a_{k+i}(n)}{a_k(n-i)} & i \leq m-k \\ 0 & i > m-k \end{cases} \tag{36}$$

$$b_i'(n) = \frac{b_i(n-k)}{a_k(n-i)} \tag{37}$$

where $b_0(n) = 1$ and $a_i(n) = 0$ for $i < k$.

Proof: The equations for the circuit of Fig. 1 can be written

$$q_i(n) = q_1(n-i+1) \qquad 1 < i \leq m \tag{38}$$

$$q_1(n) = x(n) - \sum_{i=1}^{m} b_i(n) q_1(n-i) \tag{39}$$

$$y(n) = \sum_{i=k}^{m} a_i(n) q_1(n-i). \tag{40}$$

Similarly, the equations of the system of Fig. 2 can be written

$$q_i'(n) = q_1'(n-i+1) \qquad 1 < i \leq m \tag{41}$$

$$q_1'(n) = y(n) - \sum_{i=1}^{m} a_i'(n) q_1'(n-i) \tag{42}$$

$$z(n) = \sum_{i=0}^{m} b_i'(n)q_1'(n-i) \tag{43}$$

Rewriting (40) we have

$$y(n) = \sum_{i=0}^{m-k} a_{k+i}(n)q_1(n-i-k). \tag{44}$$

Substituting (36) and (44) into (42) produces

$$0 = \sum_{i=0}^{m-k} a_{k+i}(n)\left[q_1(n-i-k) - \frac{q_1'(n-i)}{a_k(n-i)}\right]. \tag{45}$$

Therefore

$$q_1'(n-i) = a_k(n-i)q_1(n-i-k) \tag{46}$$

or

$$q_1'(n) = a_k(n)q_1(n-k). \tag{47}$$

Substituting (47) and (37) into (43) yields

$$z(n) = \sum_{i=0}^{m} \frac{b_i(n-k)}{a_k(n-i)} a_k(n-i)q_1(n-k-i) \tag{48}$$

$$= \sum_{i=0}^{m} b_i(n-k)q_1(n-k-i). \tag{49}$$

Equation 39 can be rewritten

$$x(n) = \sum_{i=0}^{m} b_i(n)q_1(n-i). \tag{50}$$

By comparing (49) and (50) it can be seen that

$$z(n) = x(n-k). \tag{51}$$

Hence, the system described by the theorem is the quasi-inverse. Not only does the quasi-inverse exist, but it is of the same form as the inverse of the fixed system and is in a canonical form.

Let us now consider some examples of the application of the theorem.

Example 1.

$$\begin{aligned} h_0(n) &= \tfrac{1}{2}[1 + (-1)^n] \\ h_1(n) &= (\tfrac{1}{2})^n \\ h_2(n) &= 1. \end{aligned} \tag{52}$$

The transmission matrix corresponding to (52) is given by

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 1 & \frac{1}{2} & 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & \frac{1}{4} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & \frac{1}{8} & 1 & 0 & \cdots \\ \vdots & 0 & 0 & 1 & \frac{1}{16} & 0 & \cdots \\ & \vdots & 0 & 0 & 1 & \cdot & \\ & & \vdots & 0 & 0 & \cdot & \cdot \\ & & & \vdots & 0 & & \\ & & & & \vdots & & \cdot \end{bmatrix}. \tag{53}$$

It is necessary now to apply Theorem 1 to decide whether or not a quasi-inverse exists. For $i = 1$, the theorem is satisfied for $k \geq 0$, that is, an inverse with delay zero exists. Let us now consider $i = 2$. For $k = 1$, we have

$$\mathbf{H}_{i+k} = \mathbf{H}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & \frac{1}{2} & 1 \end{bmatrix}. \tag{54}$$

First, the rank of $\mathbf{H}_{21}$ is two since $\begin{vmatrix} 1 & 0 \\ 1 & \frac{1}{2} \end{vmatrix} \neq 0$, which satisfies condition one of the theorem. The rank of $\mathbf{H}_3$ is two. However $r' = 1$ which is greater than $r - i = 0$, violating the second condition of the theorem. It is next necessary to consider $k = 2$ for $i = 2$. In this case

$$\mathbf{H}_{i+k} = \mathbf{H}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & \frac{1}{2} & 1 & 0 \\ 0 & 1 & \frac{1}{4} & 0 \end{bmatrix}. \tag{55}$$

For this, the rank of $\mathbf{H}_{22}$ is two, which equals i; the rank of $\mathbf{H}_{22}'$ is one; and the rank of $\mathbf{H}_4$ is three. This satisfies the theorem for $i = 2$. This process can be continued and it can be shown that the theorem is satisfied for $k = 2$.

This may readily be seen by solving the equations represented by the matrix in an iterative manner. It can be seen that each row of the quasi-inverse matrix can be solved for independently. Appendix II indicates the method of solution for the first eight rows of the quasi-inverse

$$
\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
1 & 0 & & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\
2 & 0 & & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\
3 & 1 & & 0 & 0 & 0 & 0 & 0 & 0 \cdots \\
4 & \frac{2}{7} & & -\frac{2}{7} & \frac{8}{7} & 0 & 0 & 0 & 0 \cdots \\
5 & -\frac{8}{7} & & \frac{8}{7} & -\frac{4}{7} & 0 & 0 & 0 & 0 \cdots \\
6 & -\frac{64}{7\cdot 127} & & \frac{64}{7\cdot 127} & -\frac{32}{7\cdot 127} & -\frac{8}{127} & \frac{128}{127} & 0 & 0 \cdots \\
7 & -\frac{8\cdot 128}{7\cdot 127} & & \frac{8\cdot 128}{7\cdot 127} & \frac{4\cdot 128}{7\cdot 127} & \frac{128}{127} & -\frac{16}{127} & 0 & 0 \cdots \\
8 & \vdots & & \vdots & & & & -\frac{32}{2047} & \frac{2048}{2047} \\
 & & & & & & & \vdots & \vdots
\end{array}
\tag{56}
$$

where only the sum of the first two columns is indicated, since only this sum is determined.

It can be seen from Theorem 2 that the quasi-inverse is not unique in this example. This is evident from an examination of the solution. The first two columns of the quasi-inverse matrix are not fully determined. Only their sum is fixed.

Example 2.

As a second example, consider the case of Fig. 1 where $a_0(n) \neq 0$ for all n and $m = 2$. If we choose $a_i(n)$ and $b_i(n)$ as

$$
\left.
\begin{aligned}
a_0 &= 1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8} \cdots \\
a_1 &= 1, 0, 1, 0 \cdots \\
a_2 &= 1, 1, 1, 1 \cdots \\
b_1 &= 1, 2, 3, 4 \cdots \\
b_2 &= 1, 0, 1, 0 \cdots
\end{aligned}
\right\} \tag{57}
$$

then we can make use of (36) and (37) and find

$$
\left.
\begin{aligned}
a_1' &= \phi, 0, 2, 0, 8, 0, 32 \cdots \\
a_2' &= \phi, \phi, 1, 2, 4, 8, 16 \cdots \\
b_0' &= 1, 2, 4, 8, 16, 32, 64 \cdots \\
b_1' &= \phi, 2, 6, 16, 40, 96, 224 \cdots \\
b_2' &= \phi, \phi, 1, 0, 4, 0, 16 \cdots
\end{aligned}
\right\} \tag{58}
$$

where the ϕ's are undetermined terms since no input can get through one delay until after one unit of time and through two delays until after two units of time. These results can be verified by finding the signal at each unit of time at each point in the circuit.

IV. CONCLUSION

This paper has presented a set of necessary and sufficient conditions for the existence of a quasi-inverse of a discrete time-varying system. The conditions are straightforward and mathematically simple to apply. However, in the general case, an infinite number of calculations are required. The existence of a quasi-inverse of a finite string of input symbols can be determined in a finite number of steps although a computer may be necessary. Corollaries were presented with more readily applicable conditions for some special cases. The theorems are useful to prove other easily applied conditions for special cases that might be useful in other situations.

In addition a theorem was proved for finite transmission matrices to decide whether a quasi-inverse exists. This theorem is useful for coding networks and applies to galois fields as well as the more familiar number system.

The realization of an actual network configuration for the quasi-inverse is a far more difficult problem than the determination of the existence of the quasi-inverse matrix. The solution has been obtained in Section III for the special case where the major diagonal (or the first non-zero diagonal) contains only non-zero terms. The inverse is then of the same form as the inverse of the time-invariant system.

Problems that remain unsolved are (1) necessary and sufficient conditions for the existence of an inverse for other important special cases, such as that in which $h_0(n)$ is periodically zero and the other $h_i(n)$ are also periodic or constant; (2) the form of the network for the general case of the inverse; and (3) a simple formulation of theorem 1 in terms of the $a_i(n)$ $b_i(n)$ parameters of Figs. 1 and 2, rather than the $h_i(n)$ parameters used primarily in Section II.

Acknowledgment

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APPENDIX I

Proof of Lemmas

Lemma 1. Given a square matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{B} \\ \mathbf{I} & \mathbf{D} \end{bmatrix}$$

where $\mathbf{I}$ is the $n \times n$ identity matrix and $\mathbf{D}$ is an arbitrary $r \times r$ matrix, the rank of $\mathbf{A}$,

$$R(\mathbf{A}) = R(\mathbf{I}) + R(\mathbf{B})$$

Proof: Using only the elementary operations (8), (that is, operations that do not change the rank of a matrix) $\mathbf{A}$ may be written in the form

$$\mathbf{A}' = \begin{bmatrix} \mathbf{I} & \mathbf{D} \\ \mathbf{O} & \mathbf{B} \end{bmatrix}$$

and $\mathbf{B}$ may be written as

$$\mathbf{B} = \begin{bmatrix} \mathbf{I}_a & \mathbf{B}' \\ \mathbf{O} & \mathbf{O} \end{bmatrix}$$

where $\mathbf{B}$ is of rank a and $\mathbf{I}_a$ is an $a \times a$ identity matrix. $\mathbf{A}'$ can then be written as

$$\mathbf{A}'' = \begin{bmatrix} \mathbf{I} & \mathbf{D}_1 & \mathbf{D}_2 \\ \mathbf{O} & \mathbf{I}_a & \mathbf{B}' \\ \mathbf{O} & \mathbf{O} & \mathbf{O} \end{bmatrix}$$

where the rank of $\mathbf{A}''$ equals the rank of $\mathbf{A}$. But the rank of $\mathbf{A}''$ is the rank of $\mathbf{I}$, n, plus the rank of $\mathbf{I}_a$, a, since there are $n + a$ non-zero rows and the submatrix

$$\begin{bmatrix} \mathbf{I} & \mathbf{D}_1 \\ \mathbf{O} & \mathbf{I}_a \end{bmatrix}$$

is non-singular.

Lemma 2. Consider a set of r independent equations in $i + k > r$ unknowns. After completion of the Gauss-Jordan reduction, they may be written as

$$\begin{aligned}
x_1 &= c_1 + \alpha_{11}x_{r+1} + \cdots + \alpha_{1,i+k-r}x_{i+k} \\
x_2 &= c_2 + \alpha_{21}x_{r+1} + \cdots + \alpha_{2,i+k-r}x_{i+k} \\
&\text{---------- (dashed)} \\
x_i &= c_i + \alpha_{i1}x_{r+1} + \cdots + \alpha_{i,i+k-r}x_{i+k} \\
&\text{---------- (dashed)} \\
x_r &= c_r + \alpha_{r1}x_{r+1} + \cdots + \alpha_{r,i+k-r}x_{i+k}
\end{aligned}$$

The necessary and sufficient condition for the existence of a unique solution for $x_1, x_2, \cdots, x_i$ is that $\alpha_{pq} = 0$ for $1 \leq p \leq i$.

Proof: (*a*) Sufficiency: If the α's are zero, then a unique solution, $x_p = c_p$ does exist for $1 \leq p \leq i$.

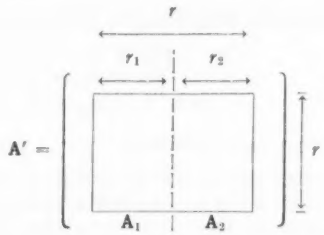
(*b*) Necessity: Assume some α_{pq} is not zero. This equation would then be

$$x_p = c_p m + \alpha_{pq}x_{q+r}.$$

It may be possible to eliminate x_{q+r} from this equation, but only by introducing another equation. If this equation is one of the first i, then there would be i equations in $i + 1$ unknowns. If this were one of the last $r - i$ equations, there would result a set of $i + 1$ equations in $i + 2$ unknowns (since equation j introduces x_j as a variable).

Lemma 3. If a matrix $\mathbf{A}$ is partitioned into two submatrices such that $\mathbf{A} = [\mathbf{A}_1 \ \mathbf{A}_2]$, then the rank of $\mathbf{A}$, $R(\mathbf{A}) \leq R(\mathbf{A}_1) + R(\mathbf{A}_2)$.

Proof: By rearranging the rows of $\mathbf{A}$ and the columns of $\mathbf{A}_1$ and $\mathbf{A}_2$ (so that no column is moved from $\mathbf{A}_1$ to $\mathbf{A}_2$) it is possible to place one of the $r \times r$ non-zero determinants (where $R(\mathbf{A}) = r$) in the r columns of $\mathbf{A}$ and in r adjacent rows, such as



The rearranging of rows and columns has not changed the rank of $\mathbf{A}$, $\mathbf{A}_1$ or $\mathbf{A}_2$. But the value of the indicated $r \times r$ determinant may be found from an expansion about $r_1 \times r_1$ determinants, all of which are in $\mathbf{A}_1$. Since the $r \times r$ determinant is non-zero, at least one of these $r_1 \times r_1$ determinants is non-zero and thus the rank of $\mathbf{A}_1$ is greater than or equal to r_1. Similarly, an expansion about $r_2 \times r_2$ determinants, all of which are in $\mathbf{A}_2$, may be made. Thus, the rank of $\mathbf{A}_2$ must be greater than or equal to r_2. Therefore

$$r_1 + r_2 = r = R(\mathbf{A}) \leq R(\mathbf{A}_1) + R(\mathbf{A}_2).$$

APPENDIX II

Example

It is desired to calculate the quasi-inverse of the system represented by the transmission matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 1 & \frac{1}{2} & 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 1 & \frac{1}{8} & 1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \frac{1}{16} & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1 & \frac{1}{32} & 1 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{64} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$

The quasi-inverse system is defined by the transmission matrix $\mathbf{H}'$ where they have an infinite number of rows and columns.

$$\mathbf{H}'\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$

If the elements of $\mathbf{H}'$ are denoted by x_{ij} where $x_{ij} = 0$ when $j > i$, a set of equations may be found for any row as follows:

$$\text{Row 1:} \quad x_{11} = 0$$

$$\text{Row 2:} \quad x_{21} + x_{22} = 0$$

$$0x_{22} = 0$$

$$\text{Row 3:}\quad \begin{aligned} x_{31} + x_{32} + x_{33} &= 1 \\ \tfrac{1}{2}x_{33} &= 0 \\ x_{33} &= 0 \end{aligned}$$

$$\text{Row 4:}\quad \begin{aligned} x_{41} + x_{42} + x_{43} &= 0 \\ \tfrac{1}{2}x_{43} + x_{44} &= 1 \\ x_{43} + \tfrac{1}{4}x_{44} &= 0 \\ 0x_{44} &= 0 \end{aligned}$$

$$\text{Row 5:}\quad \begin{aligned} x_{51} + x_{52} + x_{53} &= 0 \\ \tfrac{1}{2}x_{53} + x_{54} &= 0 \\ x_{53} + \tfrac{1}{4}x_{54} + x_{55} &= 1 \\ \tfrac{1}{8}x_{55} &= 0 \\ x_{55} &= 0 \end{aligned}$$

The equations for the remaining rows may be found in a similar manner. Notice that the first two columns appear only as a sum of the terms in those columns, that is, the only appearance of the terms x_{i1} and x_{i2} is in the first equation of each set where they always appear as $x_{i1} + x_{i2}$. Thus a unique quasi-inverse does not exist. Each set of equations may be solved simply starting with the last equation, containing only x_{ii}, for i odd, or the last two equations containing x_{ii} and $x_{i,i-1}$ for i even.

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CREEP COLLAPSE OF CYLINDRICAL SHELLS *

BY

THEIN WAH [1]

ABSTRACT

This paper develops a theory for the collapse of cylindrical shells under steady-state creep and under external radial pressure and high temperature (300 to 500 F.). The boundary conditions assumed for the cylinders approximate the "simply supported" and "fixed-end" conditions and the theory takes into account the initial deviation from circularity of the cylinder cross section. The paper thus represents an extension of the investigation described in (**5**),[2] in which cylinders with "free" ends were considered.

The theoretical predictions of collapse time (t_{cr}) are compared with experimental results on aluminum alloy (6061-T6) cylinders and show reasonably good agreement.

I. INTRODUCTION

Recent advances in technology have necessitated a study of the behavior of certain structures under high temperature conditions that such structures would not normally encounter on earth. A body moving rapidly through air, for example, experiences aerodynamic heating which causes the temperature of the body to rise. The material of the body will then usually creep and undergo deformations that would in time render the structure functionally useless. When the structure happens to be one that could become unstable under load, such as a column for example, the creep deformations will eventually become large enough to cause collapse even when the load on the column is much smaller than what is commonly termed its "critical load." The column is then said to have reached its critical time t_{cr}.[3] Analysis shows that the critical time is a function of the load on the column as well as its initial deviation from straightness and has been the subject of much recent research. The term "creep buckling" has often been used to describe this phenomenon.

In a similar fashion the creep collapse of cylindrical shells under external pressure has also become the subject of recent investigations (**1**–**5**).

The collapse time of cylinders under external lateral pressure depends not only upon the magnitude of the external pressure, the temperature, and the initial deviation from circularity of the cylinder cross

* This work was performed in the course of research on a project sponsored by the Air Research and Development Command, U. S. Air Force.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] A list of symbols used in this paper appears at the end of the text.

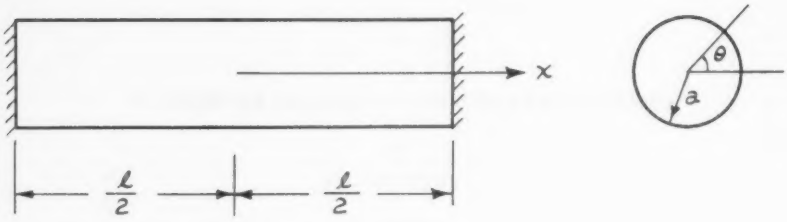

FIG. 1. Coordinate system.

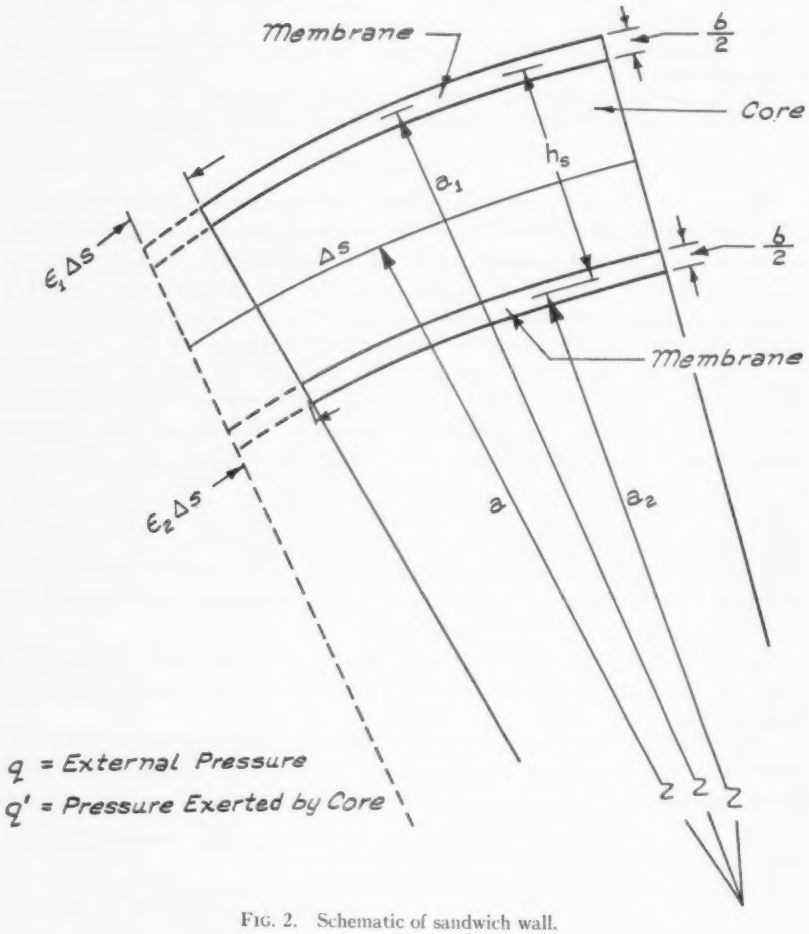

FIG. 2. Schematic of sandwich wall.

section, but also upon the boundary conditions at the ends of the cylinder. In (5) the authors have described theoretical and experimental work on such cylinders which are "free" at the ends and therefore behave very much like infinitely long tubes.

In the present paper the work is extended to cover cases of cylinders which are supported in a manner approximating "simple support" and "fixed-end" conditions. In the experimental work the simple support conditions permitted no radial displacement of the cylinder and little or no bending moment M_x at the supports (see Figs. 1 and 2). The "fixed-end" condition permitted no radial displacement and no slope $\left(\frac{\partial w}{\partial x} = 0\right)$ at the supports. On the other hand, the test arrangements permitted practically no longitudinal displacement ($u = 0$) at the ends of the cylinder in either case. Ideally, the ends of the cylinders should be free to move longitudinally but the difficulties involved in the test procedure precluded the possibility of satisfying this boundary condition. However, it is believed that the boundary conditions actually used are more realistic in the sense that they are more likely to be encountered in practice.

A rigorous analytical derivation of the collapse time (t_{cr}) for cylinders supported in the manner described above is attended with so many mathematical difficulties that the attempt had to be abandoned in favor of an approximate solution based on several simplifying assumptions. The approximate formula so devised agrees fairly well with experimental work.

II. BASIC ASSUMPTIONS

Assumptions had to be made concerning both the geometry of the cylinder as well as the creep law. The major assumptions are as follows:

1. The solid wall of the tube can be replaced by a sandwich wall of a special type (Fig. 2). The inner and outer sheets resist only membrane stresses whereas the core material takes no normal stress either in the circumferential direction or in the longitudinal direction while taking shear stresses without distortion.
2. A special property of the wall is that all the strain in the direction of the thickness takes place in the core and none whatever in the facings.
3. The uniaxial creep law developed by Stowell (6) can be extended to the case of multiaxial stress by using the concept of "effective stress" and "effective strain" from plasticity theory.

III. EQUATIONS OF EQUILIBRIUM

As the outer and inner faces of the sandwich are regarded as membranes, one may write the equations of equilibrium from the membrane

theory of shells. Using the subscript 1 for the outer membrane and 2 for the inner membrane, these equations are (see Fig. 2)

$$\left.\begin{aligned} \frac{\sigma_{\theta 1}}{a_1} &= \frac{2(q - q')}{b} \\ \frac{\partial \sigma_{x1}}{\partial x} + \frac{1}{a_1}\frac{\partial \tau_{x\theta 1}}{\partial \theta} &= 0 \\ \frac{1}{a_1}\frac{\partial \sigma_{\theta 1}}{\partial \theta} + \frac{\partial \tau_{x\theta 1}}{\partial x} &= 0 \end{aligned}\right\} \tag{1}$$

$$\left.\begin{aligned} \frac{\sigma_{\theta 2}}{a_2} &= \frac{2q'}{b} \\ \frac{\partial \sigma_{x2}}{\partial x} + \frac{1}{a_2}\frac{\partial \tau_{x\theta 2}}{\partial \theta} &= 0 \\ \frac{1}{a_2}\frac{\partial \sigma_{\theta 2}}{\partial \theta} + \frac{\partial \tau_{x\theta 2}}{\partial x} &= 0 \end{aligned}\right\} \tag{2}$$

Elimination of $\tau_{x\theta 1}$ and $\tau_{x\theta 2}$ from Eqs. 1 and 2, respectively, leads to the systems

$$\left.\begin{aligned} \frac{\sigma_{\theta 1}}{a_1} &= \frac{2(q - q')}{b} \\ \frac{\partial^2 \sigma_{x1}}{\partial x^2} &= \frac{1}{a_1^2}\frac{\partial^2 \sigma_{\theta 1}}{\partial \theta^2} \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} \frac{\sigma_{\theta 2}}{a_2} &= \frac{2q'}{b} \\ \frac{\partial^2 \sigma_{x2}}{\partial x^2} &= \frac{1}{a_2^2}\frac{\partial^2 \sigma_{\theta 2}}{\partial \theta^2} \end{aligned}\right\} \tag{4}$$

The equations of equilibrium of the sandwich wall itself are

$$\left.\begin{aligned} \sigma_{\theta 1} &= \bar{\sigma}_\theta + \frac{2M_\theta}{bh_s} \\ \sigma_{\theta 2} &= \bar{\sigma}_\theta - \frac{2M_\theta}{bh_s} \\ \sigma_{x1} &= \bar{\sigma}_x + \frac{2M_x}{bh_s} \\ \sigma_{x2} &= \bar{\sigma}_x - \frac{2M_x}{bh_s} \end{aligned}\right\} \tag{5}$$

in which

$$\left.\begin{aligned} \bar{\sigma}_\theta &= \frac{\sigma_{\theta 1} + \sigma_{\theta 2}}{2} \\ \bar{\sigma}_x &= \frac{\sigma_{x1} + \sigma_{x2}}{2} \end{aligned}\right\} \tag{6}$$

If one lets

$$\left.\begin{aligned} a_1 &= a + \frac{h_s}{2} - w \\ a_2 &= a - \frac{h_s}{2} - w \end{aligned}\right\} \tag{7}$$

one obtains from the first of Eqs. 3 and 4

$$\frac{\sigma_{\theta 1}}{a + \frac{h_s}{2} - w} + \frac{\sigma_{\theta 2}}{a - \frac{h_s}{2} - w} = \frac{2q}{b}. \tag{8}$$

An expression for the average stress $\bar{\sigma}_\theta = \frac{\bar{\sigma}_{\theta 1} + \bar{\sigma}_{\theta 2}}{2}$ may be obtained by considering a shell of radius a and thickness b loaded with a pressure q

$$\bar{\sigma}_\theta = \frac{\sigma_{\theta 1} + \sigma_{\theta 2}}{2} = \frac{qa}{b}, \tag{9}$$

from which

$$\sigma_{\theta 2} = \frac{2qa}{b} - \sigma_{\theta 1}. \tag{10}$$

Substituting Eqs. 9 and 10 into Eq. 8 and neglecting terms of the order h_s^2 as compared to terms of the order ah_s one finds

$$\left.\begin{aligned} \sigma_{\theta 1} &= \frac{qa}{b} + \frac{2qa}{b}\frac{w}{h_s} \\ \sigma_{\theta 2} &= \frac{qa}{b} - \frac{2qa}{b}\frac{w}{h_s} \end{aligned}\right\} \tag{11}$$

Combination of Eq. 11 with the first two of Eqs. 5 leads to

$$\sigma_{\theta 1} - \sigma_{\theta 2} = \frac{4qa}{b}\frac{w}{h_s} = \frac{4M_\theta}{bh_s}$$

or

$$M_\theta = qaw. \tag{12}$$

From the second of Eqs. 3 and 4 one may derive

$$\frac{\partial^2}{\partial x^2}(\sigma_{x1} + \sigma_{x2}) = \frac{1}{a_1^2}\frac{\partial^2 \sigma_{\theta 1}}{\partial \theta^2} + \frac{1}{a_2^2}\frac{\partial^2 \sigma_{\theta 2}}{\sigma \theta^2}$$

$$\frac{\partial^2}{\partial x^2}(\sigma_{x1} - \sigma_{x2}) = \frac{1}{a_1^2}\frac{\partial^2 \sigma_{\theta 1}}{\partial \theta^2} - \frac{1}{a_2^2}\frac{\partial^2 \sigma_{\theta 2}}{\sigma \theta^2}.$$

One may simplify these by replacing a_1^2 and a_2^2 by a^2, and then using Eqs. 5 to find

$$\left.\begin{aligned} \frac{\partial^2 \bar{\sigma}_x}{\partial x^2} &= \frac{1}{a^2}\frac{\partial^2 \bar{\sigma}_\theta}{\partial \theta^2} \\ \frac{\partial^2 M_x}{\partial x^2} &= \frac{1}{a^2}\frac{\partial^2 M_\theta}{\partial \theta^2} \end{aligned}\right\} \tag{13}$$

Noting from (9) that $\bar{\sigma}_\theta$ is a constant the first of Eqs. 13 may be integrated twice to yield

$$\bar{\sigma}_x = xF_1(\theta) + F_2(\theta).$$

If x is measured from the center cross section of the shell, symmetry demands that $F_1(\theta) = 0$. Thus

$$\bar{\sigma}_x = F_2(\theta). \tag{14}$$

The results so far derived are summarized below

$$\left.\begin{aligned} \bar{\sigma}_\theta &= \frac{qa}{b} \\ M_\theta &= qaw \\ \bar{\sigma}_x &= F_2(\theta) \\ \frac{\partial^2 M_x}{\partial x^2} &= \frac{1}{a^2}\frac{\partial^2 M_\theta}{\partial \theta^2} \end{aligned}\right\} \tag{15}$$

In Eqs. 15, $F_2(\theta)$ is as yet undetermined. The last of these equations will not be used because its integration is dependent on the form of the deflection function w. Some comments on this equation will be made in a later section.

IV. A CREEP LAW FOR MULTIAXIAL STRESS

Stowell's formulation of the law of steady creep for uniaxial stress is

$$\dot{\epsilon} = \dot{\epsilon}' + \dot{\epsilon}''$$

or

$$\dot{\epsilon} = \frac{d}{dt}\left(\frac{\sigma}{E}\right) + 2cTe^{-\Delta H/RT}\sinh\frac{\sigma}{\sigma_o} \tag{16}$$

in which

ϵ' = elastic strain
ϵ'' = viscous strain
ϵ = total strain
E = elastic tensile modulus, psi.
σ = stress, psi.
T = temperature, ° K
ΔH = activation energy, calories per mol
R = gas constant, taken as 2 calories/mol/° K
t = time, and dot represents differentiation with respect to time
σ_o = constant, psi.

Equation 16 is generalized for multiaxial stress to read

$$\dot{\epsilon}_1 = \frac{d}{dt}\frac{1}{E}\{\sigma_1 - \nu(\sigma_2 + \sigma_3)\} + 3cTe^{-\Delta H/RT}\left(\frac{S_1}{\sigma_i}\right)\sinh\frac{\sigma_i}{\sigma_o}, \tag{17}$$

for the strain rate in the 1 direction and in which

$$\left.\begin{aligned} S_1 &= \sigma_1 - \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}, \text{ the stress deviator} \\ \sigma_i &= \frac{1}{\sqrt{2}}\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}^{1/2}, \text{ the effective stress} \end{aligned}\right\} \tag{18}$$

It is easily verified that when $\sigma_2 = \sigma_3 = 0$, formula 17 reduces to formula 16 and that if the effective viscous strain rate is defined as

$$\dot{\epsilon}_i'' = \frac{\sqrt{2}}{3}\{(\dot{\epsilon}_1'' - \dot{\epsilon}_2'')^2 + (\dot{\epsilon}_2'' - \dot{\epsilon}_3'')^2 + (\dot{\epsilon}_3'' - \dot{\epsilon}_1'')^2\}^{1/2}$$

then

$$\epsilon_i'' = 2cTe^{-\Delta H/RT}\sinh\frac{\sigma_i}{\sigma_o}.$$

That is, the effective viscous strain rate is related to the effective stress in the same way as the viscous strain rate is to the stress in the uniaxial case.

Applying Eq. 17 to the membranes in the sandwich wall, one obtains

$$\left.\begin{aligned}\epsilon_{\theta 1} &= \frac{d}{dt}\frac{1}{E}\{\sigma_{\theta 1} - \nu\sigma_{x1}\} + 3cTe^{-\Delta H/RT}\frac{S_{\theta 1}}{\sigma_{i1}}\sinh\frac{\sigma_{i1}}{\sigma_o}\\ \epsilon_{\theta 2} &= \frac{d}{dt}\frac{1}{E}\{\sigma_{\theta 2} - \nu\sigma_{x2}\} + 3cTe^{-\Delta H/RT}\frac{S_{\theta 2}}{\sigma_{i2}}\sinh\frac{\sigma_{i2}}{\sigma_o}\end{aligned}\right\} \tag{19}$$

Using Eqs. 5 and 18 it is readily shown that

$$\left.\begin{aligned}S_{\theta 1} &= \frac{1}{3}\left\{2\bar{\sigma}_\theta - \bar{\sigma}_x + \frac{2}{bh_s}(2M_\theta - M_x)\right\}\\ S_{\theta 2} &= \frac{1}{3}\left\{2\bar{\sigma}_\theta - \bar{\sigma}_x - \frac{2}{bh_s}(2M_\theta - M_x)\right\}\\ S_{x1} &= \frac{1}{3}\left\{2\bar{\sigma}_x - \bar{\sigma}_\theta + \frac{2}{bh_s}(2M_x - M_\theta)\right\}\\ S_{x2} &= \frac{1}{3}\left\{2\bar{\sigma}_x - \bar{\sigma}_\theta - \frac{1}{bh_s}(2M_x - M_\theta)\right\}\end{aligned}\right\} \tag{20}$$

and

$$\left.\begin{aligned}\sigma_{i1} &= \{\sigma_{\theta 1}^2 + \sigma_{x1}^2 - \sigma_{\theta 1}\sigma_{x1}\}^{1/2}\\ \sigma_{i2} &= \{\sigma_{\theta 2}^2 + \sigma_{x2}^2 - \sigma_{\theta 2}\sigma_{x2}\}^{1/2}\end{aligned}\right\} \tag{21}$$

V. INTEGRATION OF THE EQUATIONS

From Eqs. 19 the curvature rates may be derived and one may thereby obtain a relation between the curvature rate and the stress resultants in the sandwich tube. The integration of the resulting equations is, however, extremely difficult, primarily because no simple deflection function w can be found that satisfies the boundary conditions as well as being "reasonable." The shape assumed by the tube at the initial application of pressure is that of a slightly out-of-round cylinder which is subjected to a uniform lateral pressure. At $t = t_{cr}$ the shape assumed by the tube is that of the same tube as if it had been subjected to its elastic buckling pressure q_{cr}. It is well known, however, that this shape is quite different from that of a perfectly circular tube which is subjected to a uniform pressure $q < q_{cr}$. The difficulty arises mainly in guessing at the x-dependence of the mode shape.

A drastic simplification of the equations results if one makes use of assumption (2) listed under Basic Assumptions. This is equivalent to the statement

$$\left.\begin{aligned}S_{\theta 1} + S_{x1} &= 0\\ S_{\theta 2} + S_{x2} &= 0\end{aligned}\right\} \tag{22}$$

From Eqs. 20 and 22 one may derive

$$(\bar{\sigma}_\theta + \bar{\sigma}_x) = 0$$
$$(M_\theta + M_x) = 0$$

or

$$\left.\begin{aligned} \bar{\sigma}_x &= -\bar{\sigma}_\theta \\ M_x &= -M_\theta \end{aligned}\right\} \tag{23}$$

Therefore

$$\left.\begin{aligned} S_{\theta 1} &= \bar{\sigma}_\theta + \frac{2M_\theta}{bh_s} \\ S_{\theta 2} &= \bar{\sigma}_\theta - \frac{2M_\theta}{bh_s} \\ \sigma_{i1} &= \sqrt{3}\left(\bar{\sigma}_\theta + \frac{2M_\theta}{bh_s}\right) \\ \sigma_{i2} &= \sqrt{3}\left(\bar{\sigma}_\theta - \frac{2M_\theta}{bh_s}\right) \end{aligned}\right\} \tag{24}$$

Substitution of Eqs. 24 into Eqs. 19 yields

$$\dot{\epsilon}_{\theta 1} = \frac{\dot{\sigma}_{\theta 1}}{E} - \frac{\nu\dot{\sigma}_{z1}}{E} + \sqrt{3}\xi \sinh\left\{\sqrt{3}\left(\sigma_\theta + \frac{2M_\theta}{bh_s}\right)\Big/\sigma_o\right\}$$

$$\dot{\epsilon}_{\theta 2} = \frac{\dot{\sigma}_{\theta 2}}{E} - \frac{\nu\dot{\sigma}_{z2}}{E} + \sqrt{3}\xi \sinh\left\{\sqrt{3}\left(\sigma_\theta - \frac{2M_\theta}{bh_s}\right)\Big/\sigma_o\right\}$$

in which $\xi = cTe^{-\Delta H/RT}$. Substracting and dividing by h_s, one may derive

$$-\dot{\kappa}_\theta = \frac{\dot{\epsilon}_{\theta 1} - \dot{\epsilon}_{\theta 2}}{h_s} = \frac{4\dot{M}_\theta}{Ebh_s^2}(1+\nu) + \frac{2\sqrt{3}\xi}{h_s}\sinh\frac{2\sqrt{3}M_\theta}{\sigma_o bh_s}\cosh\frac{\sqrt{3}\bar{\sigma}_\theta}{\sigma_o} \tag{25}$$

in which κ_θ is the curvature in the θ direction.

In deriving an expression for the curvature it is necessary to assume a form for the displacement functions w and v. Taking v as identically zero, let

$$w = h_s f \cos n\theta\phi(x). \tag{26}$$

In Eq. 26, f is a nondimensional function of time such that at $t = 0$, $f = f_0$. n is the number of "lobes" in which the tube would collapse if the elastic buckling pressure q_{cr} were applied to it and $\phi(x)$ is a function of x that satisfies the boundary conditions, with the origin of the x-coordinate taken at the center of the tube and such that $\phi(0) = 1$.

With $v = 0$, the curvature κ_θ becomes

$$\kappa_\theta = \frac{1}{a^2}\frac{\partial^2 w}{\partial\theta^2} = -h_s f\frac{n^2}{a^2}\cos n\theta\phi(x) \tag{27}$$

and from Eq. 15

$$M_\theta = -qahf \cos n\theta\phi(x). \tag{28}$$

The linear relation between curvature and moment evident in Eqs. 27 and 28 is the result of the neglect of all the modes except one.

Substituting Eqs. 27 and 28 into Eq. 25 and omitting the common factor $\cos n\theta\phi(x)$, one finds

$$1 = \frac{4qa^3(1+\nu)}{n^2Ebh_s^2} + \frac{2\sqrt{3}a^2\xi}{n^2h_s^2f} \sinh \frac{2\sqrt{3}\bar{\sigma}_\theta f}{\sigma_o} \cosh \sqrt{3}\frac{\bar{\sigma}_\theta}{\sigma_o}. \tag{29}$$

Defining

$$\frac{n^2Ebh_s^2}{4a^3(1+\nu)} = q_{cr}, \tag{30}$$

Eq. 29 may be written

$$\frac{df}{\sinh \dfrac{2\sqrt{3}\sigma_\theta f}{\sigma_o}} = \left\{ \frac{2\sqrt{3}a^2\xi}{n^2h_s^2} \cosh \sqrt{3}\frac{\bar{\sigma}_\theta}{\sigma_o} \right\} dt \Big/ \left(1 - \frac{q}{q_{cr}}\right).$$

Defining the critical time, t_{cr}, as that when $|f|_{\max}$ becomes indefinitely large, integration of the above leads to

$$\frac{t_{cr}}{\tau} = \left(\frac{q_{cr}}{q} - 1\right) \frac{\log \coth \sqrt{3}\dfrac{\bar{\sigma}_\theta}{\sigma_o} f_0}{\cosh \sqrt{3}\dfrac{\bar{\sigma}_\theta}{\sigma_o}} \tag{31}$$

where

$$\left.\begin{aligned} \tau &= \frac{2.3026(1+\nu)\sigma_o e^{\Delta H/RT}}{3EcT} \\ \log &= \log_{10} \end{aligned}\right\}. \tag{32}$$

VI. EXTENSION TO SOLID WALLS

The extension of Eq. 31 to solid wall tubes requires an interpretation of q_{cr} and f_0. In Eq. 30, if one notes that $\frac{bh_s^2}{4}$ is the moment of inertia of the sandwich wall, and is equivalent to $\frac{h^3}{12}$ for the solid wall, q_{cr} becomes

$$q_{cr} = \frac{n^2Eh^3}{12a^3(1+\nu)}. \tag{33}$$

This is taken as an approximation to the elastic buckling pressure of the solid wall tube under the given boundary conditions. In actual application, this approximate expression is replaced by the "true" value as given, for example, in (7).

f_0 is the initial out-of-roundness of the cylinder nondimensionalized with respect to the thickness h_s of the sandwich wall. Nondimensionalizing with respect to h, the thickness of the solid wall, one therefore has to replace f_0 by $\sqrt{3}f_0$ since

$$I_{\text{sand}} : I_{\text{solid}} = (bh^2/4) : (h_s^3/12) = 1:1$$
$$A_{\text{sand}} : A_{\text{solid}} = (b) : (h) = 1:1.$$

Thus

$$b = h$$
$$h_s = \frac{h}{\sqrt{3}}.$$

Furthermore, f_0 is the nondimensionalized out-of-roundness of the tube corresponding to the lobe number n after the application of the pressure. It is therefore necessary to make a harmonic analysis of the deviation from circularity of the cylinder circumference and pick the maximum amplitude in the n^{th} mode. This can ordinarily be accomplished only before the application of the external pressure q. As the application of the external pressure will magnify the out-of-roundness, one may write

$$f_0 = \frac{w_0}{h} = \frac{f_{00}}{1 - \frac{q}{q_{cr}}} = \frac{w_{00}}{h\left(1 - \frac{q}{q_{cr}}\right)} \tag{34}$$

where w_{00} is the amplitude of the out-of-roundness measured in, say, inches, before the application of the pressure.

Thus finally, the formula (Eq. 31) for a solid wall tube appears in the form

$$\frac{t_{cr}}{\tau} = \left(\frac{q_{cr}}{q} - 1\right) \frac{\log \coth 3 \frac{\bar{\sigma}_\theta}{\sigma_o} f_0}{\cosh \sqrt{3} \frac{\bar{\sigma}_\theta}{\sigma_o}} \tag{35}$$

with f_0 and τ defined by Eqs. 34 and 32, respectively.

VII. COMMENTS ON THEORETICAL DERIVATION

It is at first surprising that Eq. 35 does not contain the length of the cylinder as a parameter. However, the length and boundary conditions

are involved in calculating q_{cr} and therefore are contained in Eq. 35 both explicitly through q_{cr} and implicitly through f_0.

Returning to Eqs. 15, it is seen that the last two equations have not been used and have in fact been replaced by the simple relations of Eqs. 23. However, comparing the third of Eqs. 15 with the first of Eqs. 23 it follows that

$$f_2(\theta) = -\bar{\sigma}_\theta. \tag{36}$$

On the other hand the last of Eqs. 15 and second of Eqs. 23 yield

$$\left.\begin{aligned} \frac{1}{a^2}\frac{\partial^2 M_x}{\partial \theta^2} + \frac{\partial^2 M_x}{\partial x^2} &= 0 \\ \frac{1}{a^2}\frac{\partial^2 M_\theta}{\partial \theta^2} + \frac{\partial^2 M_\theta}{\partial x^2} &= 0 \end{aligned}\right\} \tag{37}$$

That is, both M_x and M_θ satisfy Laplace's equation. If use is made of the second of Eqs. 15 it is seen that w satisfies Laplace's equation also. It may be shown that this is the result of using the membrane theory of shells with $\bar{\sigma}_\theta$ and $\bar{\sigma}_x$ as constant over the length of the shell and neglecting the v displacement. Clearly these cannot be justified mathematically.

TABLE I.—*Values of Material Constants for Aluminum Alloy 6061-T6 Based on Creep-Strain Tests of Material Cut from 6-in. Diameter Drawn Tube.*

Temperature, deg. F.	E, psi.	τ
300	9.50×10^6	2,518,800
325	9.40×10^6	560,877
350	9.27×10^6	138,706
375	9.14×10^6	37,181
400	9.00×10^6	10,826
425	8.81×10^6	3,403
450	8.60×10^6	1,140
475	8.33×10^6	408.7
500	8.00×10^6	155.6

$$\tau = \frac{2.3026(1+\nu)\sigma_o}{3EcT}\, e^{\Delta H/RT}$$

σ_o = constant, 2500 psi.
E = elastic tensile modulus, psi.
T = temperature, ° K
c = constant, 28.7×10^6 per hour ° K
ΔH = activation energy, 39,000 calories per mol
R = gas constant, taken as 2 calories/mol/° K
ν = Poisson's ratio = 0.3

TABLE II.—*Summary of Results.*

Cyl. No.	l, in.	a, in.	h, in.	w_{os}, in.	Temp., deg. F.	q, psi	q_{cr}, psi	t_{cr} theory, hr.	t_{cr} Exp., hr.	Lobes n Th.	Lobes n Exp.
S-1	12	2.9863	0.08139	0.0136	500	130	263.0	0.23	0.00	4	4
S-2	48	2.9916	0.08113	0.0085	400	53.9	58.4	0.53	1.200	2	2
S-3	36	2.9914	0.08156	0.0052	400	73.2	79.80	1.8	0.05	2	2
S-4	13.5	2.9955	0.08156	0.00603	500	123	222.1	1.6	0.30	3	4
S-5	15	2.9947	0.0824	0.00352	475	94	199.7	42.4	80.5	3	3
S-6	18	2.9958	0.08158	0.0121	475	70.4	164.0	31.0	28.3	3	3
S-7	27	2.9876	0.08139	0.0062	425	96.8	127.3	8.7	15.50	2	2
S-8	24	2.9863	0.08135	0.0020	425	128	144.7	2.5	0.15	3	3
S-9	21	2.9880	0.08134	0.0008	450	118	151.8	21.6	13.00	3	3
S-10	48	2.9907	0.08140	0.0039	350	60	60.8	0.0	0.0	2	2
S-11	15	2.9969	0.08139	0.0012	475	182	192.0	0.004	0.0	4	3
S-12	18	2.9944	0.08135	0.0013	475	123	162.9	5.5	7.40	3	3
F-1	22	2.9943	0.08149	0.0011	425	143.5	204.4	43.2	30.00	3	3
F-2	13	2.9948	0.08140	0.0026	475	194	334.5	1.03	2.90	4	4
F-3	25	2.9914	0.08121	0.0023	425	138	178.5	11.7	13.0	3	3
F-4	19	2.9800	0.08150	0.0026	450	169.6	252.9	3.1	5.0	3	3
F-5	34	2.9907	0.08138	0.0098	400	112.2	136.01	0.26	0.10	2	2
F-6	10	2.9966	0.08025	0.0025	500	196	422.5	0.73	0.23	5	5
F-7	16	2.9895	0.08165	0.0050	475	173	275.7	0.42	0.23	4	4
F-8	46	2.9875	0.08115	0.0110	350	74	92.9	116.9	0.03	2	2
F-9	28	2.9867	0.08138	0.0009	400	140	166.1	188.6	0.06	3	3
F-10	46	2.9870	0.08144	0.0152	350	67	93.9	368.3	0.03	2	2
F-11	28	2.9900	0.08129	0.0033	375	124*	165.2	7.8	0.00	3	2–3

NOTES:

* Failed at 112 psi.
cylinder material Al Alloy 6061-T6
a = mean (outside) radius of cylinder
F = fixed end
h = mean value of measurements at ends of cylinder
l = length of cylinder
S = simple support
w_a and w_b are measured between center and ends, w_c measured at the center
$w_{os} = \frac{1}{4}(w_a + 2w_c + w_b)$ = mean amplitude of initial out-of-roundness in the n^{th} mode
See List of Symbols for other notations.

VIII. COMPARISON WITH EXPERIMENTS

Two types of tests were carried out. The first was a series of tension-creep tests to determine the parameters involved in the creep law (Eq. 16). The results together with the assumed values of E and the derived values of τ are tabulated in Table I.

Table II gives details of the cylinder geometry and shows a comparison of experimental and theoretical results. Specific details regarding experimental procedure may be obtained from (5). The

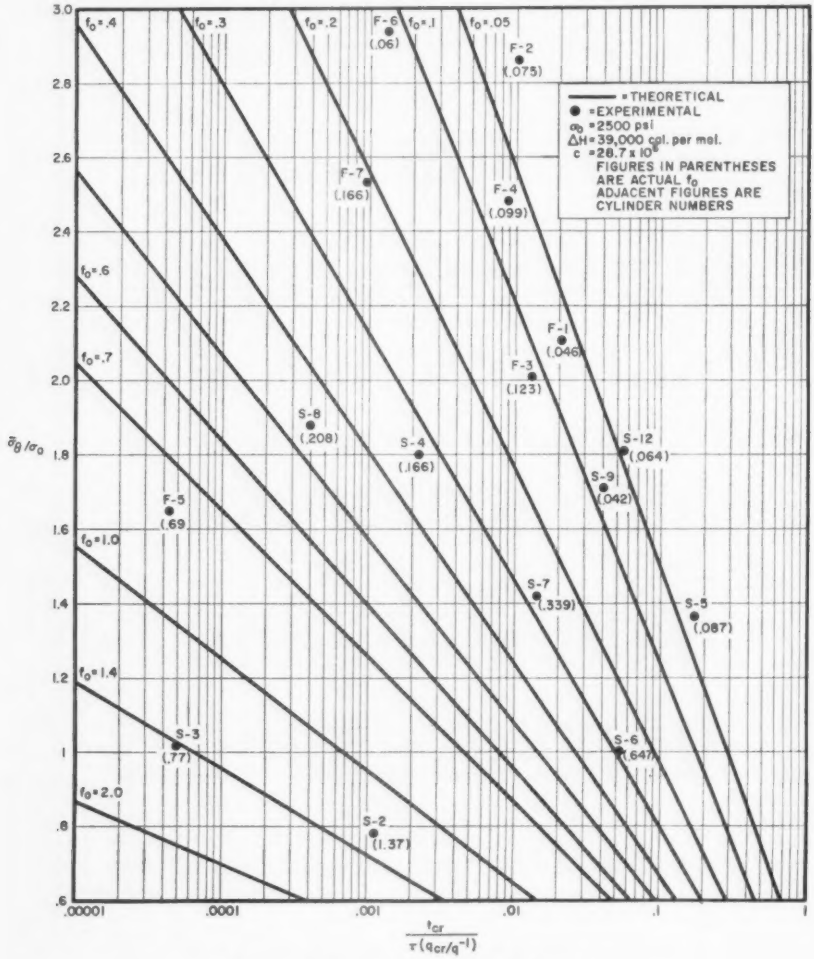

FIG. 3. Comparison of tests with theory.

NOTE: The S-6 value of (.647) shown on the figure should be (.26).

out-of-roundness of the cylinder was measured at three stations along the length of the cylinder. A harmonic analysis of the data was run and the maximum amplitude in the n^{th} mode was determined, the number n being read from the charts given on pp. 616, 617 of (7). A weighted mean of the amplitude of the out-of-roundness at the three stations was taken as the true value of w_0. The value of q_{cr} for use in formula 35 was also obtained from the charts mentioned above.

Figure 3 gives a graphical comparison of the experimental and theoretical results.

It may be noted from Table II that there is reasonably good agreement between predicted and actual values of t_{cr} except in the last four cases, which were chronologically also the last four tests carried out. The writer has been unable to account for the serious discrepancies observable in these tests. A check of the possibility of plastic action showed that the average stress $\sqrt{3}\sigma_\theta$ was in all cases far below the yield stress of the material at the temperature and time of exposure to the temperature of the test. The possibility exists that the maximum stress (bending plus average stress) exceeded the yield, but this possibility exists in all the tests and not merely in the last four.

Acknowledgment

The experimental work was carried out by R. Kirk Gregory, Research Engineer, with the assistance of Rene Guerra, and the numerical computations by Edward Briggs. The author is indebted to these colleagues for their help in the course of the research.

LIST OF SYMBOLS

a = mean radius of cylinder, in.
b = twice thickness of membrane facings of sandwich, in.
c = constant, per hour per ° K
e = natural base of logarithms
e_i = effective strain
$f = \frac{w}{h}$ = nondimensional deflection
$f_0 = \frac{w_0}{h}$ = initial nondimensional deflection at $t = 0$ after load application
$f_{00} = \frac{w_{00}}{h}$ = initial nondimensional deflection before load application
h = thickness of solid wall of cylinder, in.
h_s = thickness of sandwich wall, in.
M_θ, M_x = bending moments per unit length, lb.
n = number of circumferential lobes at buckling
q = applied pressure, psi.
q_{cr} = elastic buckling pressure, psi.
R = gas constant, taken as 2 cal/mol/° K
T = temperature, ° K
t = time, hr.
t_{cr} = collapse time, hr.

u = displacement in the x direction, in.
v = displacement in the θ direction, in.
w = displacement in the radial direction, in.
w_0 = initial deflection after load application, in.
w_{00} = initial deflection before load application, in.
ΔH = activation energy, calories per mol
ϵ = strain
ϵ' = elastic strain
ϵ'' = viscous strain
θ, x = cylindrical coordinates
κ = curvature
ν = Poisson's ratio
$\xi = cTe^{-\Delta H/RT}$
σ_i = effective stress, psi.
σ_o = constant in creep law, psi.
σ_x = normal stress in x direction, psi.
σ_θ = normal stress in θ direction, psi.
τ = temperature constant defined by Eq. 32
$\tau_{x\theta}$ = shear stress

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HIGH-PURITY TRIMETHYLBORANE

Trimethylborane of 99.9 mole per cent purity has been prepared by G. S. Ross, D. Enagonio, C. A. Hewitt, and A. R. Glasgow of the National Bureau of Standards. In this highly purified form the compound has proved useful as a neutron-capturing gas in proportional counters. It has also been used in thermodynamic studies requiring extremely high-purity samples of various substances.

Because trimethylborane ($B(CH_3)_3$) is a gaseous, toxic, reactive material that ignites on contact with air, special precautions were taken to insure safe handling. The gas, which was prepared in other laboratories, was converted to the solid, relatively stable ammonia adduct ($NH_3:B(CH_3)_3$) before being shipped to the Bureau for purification. Also, specially constructed stainless-steel apparatus was used for the liberation, distillation, and storage of the trimethylborane.

Before being reduced to trimethylborane, the adduct was purified by fractionation in a glass vacuum apparatus. Mass spectrographic analysis of each fraction permitted selection of the purest material for reduction to trimethylborane. After purification, the adduct was reacted with hydrogen chloride gas to liberate trimethylborane. Because commercial hydrogen chloride often contains hard-to-remove organic impurities, the gas was generated from sodium chloride and sulfuric acid.

Trimethylborane was released from the adduct by reaction with hydrogen chloride in the stainless steel apparatus. The reactants were held at room temperature for at least 24 hr. to insure completeness of reaction. The trimethylborane was refluxed and then distilled into a stainless-steel storage cylinder. Determination of the vapor pressure soon after preparation, and again after several weeks of storage, indicated that the gas is stable at room temperature when confined in air-tight, stainless-steel containers.

Cryoscopic analysis of the liberated trimethylborane indicated that a purity of 99.9 mole per cent had been achieved. The infrared spectra of the purified gas were obtained by J. B. Stewart of the Bureau. Several bands previously assigned to trimethylborane were totally absent, leading to the conclusion that they were due to impurities in the earlier materials.

The highly purified trimethylborane has been used as a neutron capturing gas in proportional counters by the Naval Research Laboratory. It was known that the large number of hydrogen atoms in the

$B(CH_3)_3$ molecule would effectively slow epithermal neutrons, permitting their capture by the boron nucleus. To do this, however, the gas must be in an extremely pure form. Until the Bureau developed this method of purification, trimethylborane of sufficient purity was not available for use in counters.

Measurements of many of the thermodynamic properties of trimethylborane were made by the Bureau's heat measurements laboratory. Some of the values determined are:

Equilibrium melting temperature	113.21° K.
Molal heat of fusion	3250 abs j/mole
Molal heat of transition	810 abs j/mole

Smoothed values of the molal heat capacity, enthalpy, entropy, and Gibbs free energy were computed from the data over a wide range of temperatures.

The heats of formation and combustion of trimethylborane were determined by the Bureau's thermochemistry laboratory. The values obtained are:

Heat of formation	− 34.8 kcal/mole at 25° C.
Heat of combustion (liquid)	− 714.5 kcal/mole at 25° C.

ANTENNAS FOR DETECTING MICROPULSATIONS

Special-purpose antennas for investigating micropulsations in the Earth's magnetic field have been designed and constructed by the National Bureau of Standards. These loop antennas, each $6\frac{1}{2}$ ft. in diameter and containing 32,000 turns of nylon-coated copper wire, will be used by Dr. W. H. Campbell of the Bureau's Central Radio Propagation Laboratory, Boulder, Colo., to collect data on the behavior of micropulsations and to discover how they can be related to unusual manifestations of solar or magnetic activity. Results of this program, only recently begun, are expected to provide further useful information about ionospheric disturbances arising from extraterrestrial sources which may affect radio communications.

Oscillations of the Earth's magnetic field may be divided into three general period groups: (1) The giant pulsations, often observed on standard magnetometer records, which generally have periods of about one to three minutes and amplitudes reaching ten or so gamma; (2) micropulsations with oscillation periods ranging from 5 to 30 seconds and with amplitudes reaching several gamma; and (3) oscillation periods in the range 0.5 to 3 seconds, which appear more rarely than the other two groups, with amplitudes occasionally reaching one gamma. The outstanding features of the three groups are their nearly sinusoidal form and the fact that the group characteristic periods and average amplitudes both decrease roughly by powers of ten.

On the rim of each loop antenna, in four separate channels, is wound

130 miles of copper wire. The copper wire is covered by a layer of copper screening, and finally the whole antenna is coated with Fiberglas for protection and waterproofing. This protection is necessary because the antennas will be buried in the earth to prevent interference from wind action. Sensitive enough to respond to faint, ultra low frequency electromagnetic waves or micropulsations, the antennas will be used to study oscillations in the broad range of periods from 200 sec to 0.1 sec. which can be observed on a world-wide basis.

The signals now under most intensive study by the Bureau are the oscillations of from 5- to 30-sec. periods. These micropulsations have been known to the scientific world since 1861, but little is understood about their origin or why the oscillations occur as they do. It is known that the signals are much stronger in the auroral zones and that they are probably associated with the influx of primary electrons into the ionosphere after the occurrence of solar storms.

Permanent micropulsation installations, using one antenna each, are being established near Boulder, Colorado; College, Alaska; and Huancayo, Peru. Six additional antennas will be used in the future to equip two portable stations where research will be conducted to measure and define the direction of micropulsations over broad areas.

Although the antennas will be used primarily in the study of micropulsations, they will also be of value in the study of two other types of electromagnetic waves—extra low-frequency sferics, which originate from lightning discharges, and one- to three-minute giant pulsations which are thought to be caused by the resonance of the outer atmosphere when bombarded by charged particles from the sun.

BOOK REVIEWS

HIGH-FREQUENCY MAGNETIC MATERIALS, by W. J. Polydoroff. 220 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1960. Price, $9.00.

Since the end of World War II there has been a tremendous advance in the development of new high frequency materials and in understanding their behavior. A book discussing these materials is therefore certainly welcome, because much of the relevant information cannot be found in a condensed text, but is scattered among the various scientific journals. Unfortunately this book does not fill, according to the present reviewer's opinion, the existing gap, at least with respect to the materials situation, as the author devotes only a small part of the book to important new developments in the field, and does not present an extensive enough list of references (for example, "Ferrites" by Smit and Wijn is not mentioned). Another drawback of this book is the absence of captions to figures, which makes its usage difficult.

Starting by introducing the relevant magnetic concepts the author describes briefly bulk magnetic materials. He then proceeds to a detailed discussion of the various powdered cores. Tape wound cores are however hardly mentioned. He concludes the part of the book devoted to materials with a rather brief discussion of ferrites. Here only the spinel type ferrites are considered, no mention being made of the garnets.

The second half of the book deals with applications. Here the topics dealt with are permeability tuning of transformers and ferromagnetic loop antennae. The book concludes with an account of microwave applications. In this part of the book the discussion is a more adequate one.

The present reviewer feels therefore, that in spite of its shortcomings, the book will prove to be helpful to electronic engineers interested in the field.

S. SHTRIKMAN
The Franklin Institute Laboratories

PROCEEDINGS OF THE 1960 ANNUAL INTERNATIONAL CONFERENCE ON HIGH ENERGY PHYSICS AT ROCHESTER, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos. 890 pages, diagrams, $8\frac{1}{4} \times 10\frac{3}{4}$ in. Geneva, Henri Studer, 1960. Distributed by Interscience Publishers, Inc. Price, $13.50.

The Annual International Conference on High Energy Physics was held at the University of Rochester from August 25 to September 1, 1960. It was the tenth in a series of which the first seven were held at the University of Rochester, the eighth at Cern, and the ninth at Kiev. At these meetings, leading high energy physicists in both the experimental and theoretical fields gather together to discuss the outcome of their year's work. The tenth conference was attended by approximately 350 physicists representing 30 countries.

The meeting was divided into three parts. The first part consisted of four concurrent sessions in the following categories; strong interactions of pions and nucleons (experimental), strong interactions of pions and nucleons (theoretical), strong interactions of strange particles, and weak interactions. At these sessions both invited and contributed papers were presented. The invited papers and resulting discussions are printed in full in the proceedings. Summaries by the authors of the contributed papers are printed verbatim.

The second part consisted of general sessions where rapporteurs summarized the material presented at the specialized sessions. These reports and the discussions are printed in full in the proceedings.

The last part consisted of general sessions at which invited papers of common interest were given. The sessions were on the following topics: structure of elementary particles, new results at superhigh energies, and theories of elementary particles. The proceedings contain in full these papers and the discussions.

In addition, brief summaries of all unpre-

sented papers contributed by participants are included in the proceedings. A few brief summaries of papers contributed by authors not attending the conference are also included. There is a list of all the participants and their associated laboratories. The index contains all names of authors of papers printed in the volume and participants in the discussions.

These contents of the volume make it an excellent reference for those high energy physicists who did, as well as those who did not, attend the conference. Its speedy publication in a matter of months after the conference is not the least of its merits.

DIANE F. DAVIS
Bartol Research Foundation

AN INTRODUCTION TO ASTRODYNAMICS, by Robert M. L. Baker, Jr. and Maud M. Makemson. 358 pages, diagrams, 6 × 9 in. New York, Academic Press, 1960. Price, $7.50.

The two unique and distinguishing features of this introductory textbook are its timeliness and its comprehensiveness. The authors live by the motto of Roman charity organizations: "Bis dat qui cito dat," and fill the need which existed for the last five or more years in the English technical literature in the field of Astrodynamics.

Astrodynamics is the "engineering application of celestial mechanics and allied fields such as high altitude aerodynamics, geophysics, . . . to the contemporary problems of space vehicles." The two broad divisions of the book are: *Fundamentals* (general laws, minor planets, comets, geometry, coordinate systems, astrodynamic constants) and *Detailed Analysis* (orbit determination, *n*-body problem, trajectory computation, non-gravitational forces, observation theory, etc.)

Professors Baker and Makemson are members of Dr. S. Herrick's astrodynamics school at the University of California. The book is based partially on Dr. Herrick's lectures and his ideas and beneficial influences are felt throughout it.

Proponents of the classical celestial mechanics schools will not find in this book either the elegant Hamiltonian formulations nor the usual elaborate treatments of general perturbation techniques nor an exhaustive discussion of the three body problem. The practicing engineer, on the other hand, will find the book's concise presentation and practical approach most useful.

V. G. SZEBEHELY
General Electric Co.

RADIATION DAMAGE IN SOLIDS, by Douglas S. Billington and James H. Crawford, Jr. 450 pages, diagrams, 6 × 9¼ in. Princeton, Princeton University Press, 1961. Price, $12.50.

It is indeed paradoxical that science, presumably characterized by precise definitions, occasionally assents to inaccurate nomenclature. This is a consequence of the pattern of historical development; important fields sometimes originate with the investigation of disturbing effects which interfere with the work in progress in already established areas of research. One of the many examples of a subject which has much more far-reaching implications than the name suggests is designated *radiation damage*.

The authors of this useful book have devoted their attention to one particular aspect of this broad field, namely the application of particle bombardment as a tool for introducing lattice defects into crystalline solids. They have not inappropriately retained the older term (which arose originally in connection with technological concern over the effects of bombardments on the properties of structural materials in nuclear reactors), in the light of their specific goal of discussing the effects (without regard to whether good or bad from the practical point of view) of the displacement by energetic particles of atoms from their equilibrium positions in the crystal lattice.

The tone of the treatment varies considerably throughout the book. The authors are well aware of this shortcoming, and, in fact, accept partial responsibility. Nevertheless, they might have tackled the problem somewhat more aggressively. In some sections, the approach is encyclopedic, and the authors' contribution is minimal. In others, the treatment is much more likely to fulfill one avowed purpose of the book which, it is stated, was intended to be useful to the newcomer in the field.

Chapter 1 is a general introduction describing the nature, scope, and historical development of the subject. Interactions between

radiations (including charged particles, neutrons, gamma-rays and fission fragments) are discussed in Chapter 2, and the influence of lattice imperfections on the properties of solids in Chapter 3. In accordance with the authors' contention that a treatment of radiation damage ". . . slated toward the need of the experimental solid state scientist is yet lacking," they have included a description of radiation sources in Chapter 4. Next follow successive chapters on radiation effects in metals, alloys, covalent crystals and mineral structures, ionic crystals, semi-conductors, uranium, and graphite.

There are numerous figures, and an extensive list of references is appended to each chapter in addition to a general bibliography at the end. There is no doubt that those already engaged in radiation damage investigations will regard this work as a valuable reference.

MARTIN A. POMERANTZ
Bartol Research Foundation

MODERN MATHEMATICS FOR THE ENGINEER, edited by E. F. Beckenbach. Second series, 456 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $9.50.

The rapid growth and increasing complexity of modern science and engineering exert a great pressure on the scientists and engineers to keep up-to-date with recent developments and recent techniques. It seems almost impossible to keep abreast with the progress being made even in one's own field. For these reasons, a book such as this fulfills a vital function. A group of about sixteen mathematicians and applied mathematicians specializing in diverse fields offer here skillfully written exposés introducing and explaining some of the most important recent developments in modern applied mathematics.

The book is divided in three parts, namely: 1. Mathematical Methods, 2. Statistical and Scheduling Studies, 3. Physical Phenomena. This is a well balanced partition. In the first part Erdelyi gives a comprehensive account of delta functions and distributions. Operational methods and integral transforms are treated by B. Friedman and J. Miles, semi-group methods and asymptotic formulas by R. S. Phillips and J. B. Rosser. The second part deals with information theory, the theory of control processes and of inventory processes. The main topics in the third part are functional equations in transmission line theory (R. Redheffer), characteristic value problems in magneto-hydrodynamics (S. Chandrasekhar), partial differential equation in fluid dynamics (P. Garabedian) and the numerical solution of various partial differential equations.

The style is didactic and almost uniformly good throughout. Without serious loss of vigor, appeal is frequently made to intuition and very concrete examples are treated to illustrate some of the abstract methods. The treatment of operational calculus is clear, brief and up to date; that of delta functions extremely illuminating. The last essay by Polya on "Symmetrization" is unorthodox, imaginative and delightful. Engineers and physicists will find stimulation and much useful new material in this book. Few "multi-author" books present so much unity and coherence, and the editor is to be congratulated.

HENRI AMAR
Temple University

PLASMA PHYSICS, edited by James E. Drummond. 386 pages, diagrams, plates, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $12.50.

Although, following the pioneering work of Tonks and Langmuir, plasma physics has been a recognizable field of study since about 1929, developments in this area have proceeded at an explosive rate, particularly during the last decade. The subject is of immediate interest in a wide variety of disciplines in science and technology ranging from astrophysics to electronic engineering.

Basic research on this subject is conducted not only in academic institutions, but also in many industrial organizations. Exciting possibilities, such as controlled thermonuclear fusion, or new means of propulsion of space vehicles, have fired the imagination of those concerned with the potential applications of new scientific knowledge. The microwave engineer and the cosmic ray physicist, are, like many of their colleagues in various fields of pure and applied science, now speaking the language of plasma physics.

This book provides an excellent introduction for the student or research worker who desires a sound theoretical understanding of

the basic concepts underlying this rather abstruse subject. An outgrowth of a series of seminars at the Stanford Research Institute, this valuable volume consists of contributions by eleven authors. At the behest of the editor, James E. Drummond, some, including two prominent Russian scientists, filled in gaps in the material originally covered in order to round out the book.

Following an introduction by Drummond, the book is divided into three parts: basic plasma theory, magnetohydrodynamics, and microwave plasma physics. Chapters by Y. Klimontovich and V. P. Silin on the spectra of systems of interacting particles, E. Meeron on statistical mechanics of reversible processes in plasmic dynamics, P. A. Sturrock on amplifying evanescent waves, convective and nonconvective instabilities, and O. Buneman on how to distinguish between attenuating and amplifying waves, comprise Part I. A review of magnetohydrodynamics by Y. A. Yoler, and chapters on the Bennett Pinch by O. Buneman, and stability of relativistic self-focusing streams by D. Finkelstein and P. A. Sturrock, follow in Part II. Part III treats a number of special problems including the electrical properties of high-altitude ionized shock waves by P. A. Goldberg, transverse plasma waves and plasma vortices by O. Buneman, plasma power absorption by J. E. Drummond, microwave diagnostics for controlled fusion research by C. B. Wharton, and plasma magnetron theory by J. E. Drummond. The final chapter by S. C. Brown describes the fundamental plasma physics research program at M.I.T.

Although this work was not primarily intended as a text, it could well serve as the basis for a graduate-level course. The considerable amount of new material contained in the well-written reviews of the various aspects of plasma physics will be welcomed by active workers in this important field.

MARTIN A. POMERANTZ
Bartol Research Foundation

BASIC PRINCIPLES OF FISSION REACTORS, by W. R. Harper. 314 pages, illustrations, 6 × 9 in. New York, Interscience Publishers, Inc., 1961. Price, $7.50.

The author of this excellent book has managed to present the "basic principles of fission reactors" in an especially workmanlike manner. While he discusses in general terms all of the aspects of reactor design and operation, Dr. Harper devotes a considerable portion of the text to the fundamentals of how a nuclear reactor functions and to the various analytical forms and techniques for describing and predicting this operation. This book most definitely cannot be considered a text on the engineering aspects of fission reactor design, but nevertheless it provides an exposition of the principles involved in the solution of the engineering problems, such as fuel-element design, power removal, radiation protection, and reactor control.

This book is intended as an introduction to a graduate-level course, but it also fulfills the needs of anyone who has some background in mathematics, physics and engineering for an introduction to nuclear reactors. The basic philosophy followed by the author can best be described by quoting excerpts from his preface. This reviewer believes that the author has been highly successful in following this description.

"This book takes the reader to about the stage at which detailed reactor design begins, without assuming any previous knowledge of nuclear technology. . . . The mathematical treatment has therefore been chosen to clarify basic principles . . . . Only such nuclear physics as is necessary for an understanding of reactors is given, and this is not introduced until it is required, so that it is first encountered in some reactor context . . . . It is easier to explain why a bomb explodes than how to design a power reactor so as to be safe, and it is instructive to compare the two cases. The principle of the bomb is, therefore, described in relation to the problem of taming a reactor. The dangers inherent in the peaceful uses of nuclear energy are frankly faced, so that the reader may learn how they can be properly circumvented . . . . The reactor itself is the centre-piece of the book. Attendant subjects such as radiation protection, health physics, waste disposal and economic implications are included so as to put the reactor in perspective, but are not pursued further."

In summary, this book is highly recommended for two types of individuals: someone who desires to become acquainted with the general field of nuclear reactor technology and who desires an orientation as to the termi-

nology, basic operating principles, and salient design and operational problem areas of nuclear fission reactors; and one who may be already working in one of the "engineering specialties" of the nuclear field and who desires a clearer insight into the various nuclear aspects of reactor design and operation, as well as a broader view of the entire reactor field. This book also serves to straighten out the thinking of anyone who has become confused about the fundamentals of reactor analysis from reading some of the other more extensive texts on this subject.

W. H. STEIGELMANN
The Franklin Institute Laboratories

TIME-HARMONIC ELECTROMAGNETIC FIELDS, by Roger F. Harrington. 480 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $13.50.

The strict reviewer has some misgivings upon undertaking the review of a new book on electromagnetic theory. Since the appearance around the early 1940's of the excellent texts by Stratton, Schelkunoff, Smythe, Slater, the M.I.T. Radiation Laboratory, and others, a great many recapitulations have been published. Often these have grown out of teaching notes used in the universities, a well-known process which we will not debate here. However, it is against such a background that one launches a review of an electromagnetic theory treatise; for this is an area of fundamental knowledge where real progress is slow.

After such a harsh introduction it is a relief to be able to say that Professor Harrington's book does not fall into the depressing category described above. He has included for example, sections which introduce perturbational and variational techniques. These methods are demonstrated through their use in the text for calculation of resonator quality factors and waveguide attenuation constants.

Fundamental concepts are only briefly introduced prior to a presentation of wave solutions for a variety of common transmission lines. A wire in a trough provides an interesting analysis, not usually found in texts on this subject, but of altogether practical significance.

After treating those situations which are solvable by the use of one or more plane waves and cases of sources in unbounded, homogeneous regions, a general solution to the scalar Helmholtz equation is obtained. By the method of separation of variables general solutions to the Helmholtz equation can be applied to rectangular coordinate systems, particularly waveguide problems involving this geometry. These include junctions, apertures, corrugated conductors, and various other conducting or non-conducting obstructions.

Cylindrical wave functions are developed from the Helmholtz equation to apply to circular waveguides, cavity resonators, two-dimensional radiation patterns and slot-type antennas. Free space radiation from such interesting sources as, for example, a slotted cone is analyzed in a section on spherical wave functions. Valuable, specific analytical processes making possible the maximization of antenna gain for unusual geometries are given. These are gathered from recent publications in which individuals have prepared noteworthy studies of electromagnetic field problems.

It is hard for the author not to include sections of the M.I.T. Radiation Laboratory Series in his book; because this particular source was of such a comprehensive nature. It is thus that in considering microwave networks, that is, the equivalent circuit analog of waveguides, Professor Harrington found it necessary to employ N. Marcuvitz's formulation.

The effect of bringing together past and present analytical methods in the broad topic of electromagnetic fields has been in this case splendidly executed.

C. W. HARGENS, III
The Franklin Institute Laboratories

MODERN SPACE SCIENCE, by Frederick E. Trinklein and Charles M. Huffer. 550 pages, illustrations, 6¼ × 9¼ in. New York, Rinehart and Winston, Inc., 1961. Price, $4.96.

The close tie between astronomy and space travel is being recognized more and more in our secondary schools. While in the immediate past the teaching of astronomy was held to a minimum because of the reluctance of students to take this course, today with the tremendous emphasis on space travel students are beginning to recognize the importance of astronomy to their future aspi-

rations. As a result, there is now a demand for textbooks slanted to a space oriented astronomy. Some have appeared, but these were obviously written by poorly qualified writers. These authors may have had a hold on one or the other of the two subjects, but their inability to integrate the two resulted in a mediocre presentation.

The new book, *Modern Space Science*, is a joint effort by a ranking astronomer and a highly qualified science teacher. The result of their collaboration is a superbly written book aimed at the high school student. The authors have divided their material into four sections. The first deals with the composition of the universe. Essentially, this involves the physics of the universe. The authors explore atoms, nuclear reactions, the earth and the origin and state of the universe.

The second section deals with the physical laws of space. In this section, the laws of motion and gravitation are developed. Time, light, the tools of the astronomer, and the role of space science to the verification of the theory of relativity are covered.

The third section, which the authors call the geography of space, is strictly astronomy. It is a well written, well balanced section which could serve as a text for a half-year course. The last section involves man in space. This is a rather short section but the essentials are included.

Completeness of the book is an outstanding characteristic. In the six appendices are contained the material which the authors believed essential to the course but not a necessary part of it. Thus, the use of these sections can be left to the discretion of the instructor. The book is magnificently illustrated, and at the end of each chapter is found a vocabulary review, sets of questions, problems and suggestions for further reading and work. At the end of each section, the authors have included a list of books for additional reading. *Modern Space Science* is one of the best books on this subject this reviewer has seen.

I. M. LEVITT
Fels Planetarium

PUBLICATIONS RECEIVED

PLASTICS TOOLING, by Malcolm W. Riley. Second edition, 216 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1961. Price, $7.50.

ENGINEERING DRAWING AND GEOMETRY, by Randolph P. Hoelscher and Clifford H. Springer. Second edition, v.p., illustrations, 8 × 10 in. New York, John Wiley & Sons., Inc., 1961. Price, $8.95.

COLLEGE CHEMISTRY, by Donald C. Gregg. 593 pages, illustrations, 6 × 9 in. Boston, Allyn & Bacon, Inc., 1961. Price, $7.50.

INTRODUCTION TO QUALITATIVE ANALYSIS, by Durward C. Layde. 350 pages, illustrations, 6 × 9 in. Boston, Allyn & Bacon, Inc., 1961. Price, $6.50.

PHYSICS FOR ENGINEERS AND SCIENTISTS, by Richard G. Fowler and Donald I. Meyer. Second edition, 553 pages, diagrams, 6 × 9 in. Boston, Allyn & Bacon, Inc., 1961. Price, $9.25.

SCIENCE IN SPACE. CHAPTER 1, DIMENSIONS AND PROBLEMS: A GENERAL REVIEW. A Report by the Space Science Board of the National Academy of Sciences—National Research Council. 41 pages, 8½ × 11 in. Washington, 1961. No price.

UNITS OF WEIGHT AND MEASURE. DEFINITIONS AND TABLES OF EQUIVALENTS. 62 pages, 8 × 10¼ in. Washington, U. S. Government Printing Office, 1960. Price, $0.40 (paper).

CHEMISTRY, by Michell J. Sienko and Robert Plane. Second edition, 623 pages, diagrams, plates, 6½ × 9¼ in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $7.50.

MODERN COMPUTING METHODS, by members of the staff of the Mathematics Division, National Physical Laboratory. Second edition, completely revised, 170 pages, 6 × 9½ in. London, Her Majesty's Stationery Office, 1961. Price, £1 1s.

BOOK NOTES

LINEAR ALGEBRA, by George F. Hadley. 290 pages, diagrams, 6 × 9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc., 1961. Price, $6.75.

This intermediate introduction to linear algebra is intended for a wide group of readers with no more background in mathematics than algebra and analytic geometry. It provides a good introduction to such subjects as vector analysis, matrices and determinants, and the solution of linear equations. A chapter on convex sets is written on a somewhat more sophisticated level, and the final chapter on eigenvalue problems and quadratics presents a concise introduction to a subject of importance to engineers. Many references are given, as well as a large number of practice problems.

MECHANICS, by L. D. Landau and E. M. Lifshitz, translated by J. B. Sykes and J. S. Bell. 165 pages, diagrams, 6 × 9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc.; London, Pergamon Press; 1960. Price, $6.50.

Classical Newtonian physics is treated from an advanced and rigorous point of view, on the basis of Galileo's principles of selectivity and Hamilton's principle of least action. In addition to the basic laws of mechanics, subjects discussed include particle collision problems, theory of small oscillation, rigid body motion, and the canonical equations of Hamilton. The writing is concise but exceptionally clear, and keeps the reader aware at all times of the breadth of the subject of classical mechanics which it expertly summarizes. A few problems are given with each chapter, some with solutions.

COMMUNICATION AMONG SOCIAL BEES, by Martin Lindauer. 143 pages, diagrams, illustrations, 5½ × 8¼ in. Cambridge, Harvard University Press, 1961. Price, $4.75.

In his text, Dr. Lindauer describes experiments that point the way to a deeper understanding of a subject that has long puzzled and fascinated the minds of men. The "language" by which bees communicate with one another is his subject and he covers such things as how the forager bees tell others in the hive the direction and distance to a newly discovered source of nectar; how scout bees, after being sent to find a new nesting place at the time of swarming, describe the location and quality of the place they have found. The author reports that when the swarm has decided which site is best—and it may take several days to reach agreement—the human observer who has learned the language knows just where the bees are going.

WORLD PREHISTORY—AN OUTLINE, by Grahame Clark. 284 pages, illustrations, plates, 5½ × 8½ in. New York, Cambridge University Press, 1961. Price: $6.00 (cloth); $2.45 (paper).

This brief and stimulating outline of the prehistory of the whole world and the development of the human race in each part of the globe, until the stage of detailed records from which history may be written, provides a unified framework to which bits and pieces of popular archaeological information can be referred. The book will be welcomed by an undergraduate about to start the formal study of archaeology, or a layman simply looking for an unusually interesting book.

TURBULENCE: CLASSIC PAPERS ON STATISTICAL THEORY, edited by S. K. Friedlander and Leonard Topper. 187 pages, 6 × 9¼ in. New York, Interscience Publishers, Inc., 1961. Price, $6.00.

This book is comprised of a group of papers which represent significant steps in the development of the statistical theory of turbulence. The collection should serve as a convenient reference to what is now classical literature for those interested in fluid turbulence in itself, and as a primary reference for workers in other fields, some peripheral and others rather far afield from turbulence, who find the methods developed in the statistical theory increasingly useful.

TRANSFORM CALCULUS FOR ELECTRICAL ENGINEERS, by Roger Legros and A. V. J. Martin. 342 pages, diagrams, $5\frac{3}{4} \times 9$ in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1961. Price: $12.00 (trade); $9.00 (text).

This book is divided into four parts. They cover, briefly: Part I—the authors present the Fourier Series and its applications, particularly to frequency spectra; Part II—devoted to the Laplace Transform and its applications, analyzing the subject in detail and suggesting alternative derivations; Part III—treats mainly the Laplace Transform method for the solution of linear circuits, with particular emphasis on fundamental differentiating and integrating circuits; Part IV—covers linear amplifiers and their frequency behavior, including a discussion on low- and high-frequency compensation. The authors close each section with worked out examples and problems borrowed from all branches of engineering.

THE CONSULTING ENGINEER, by C. Maxwell Stanley. 258 pages, diagrams, $5\frac{3}{8} \times 8\frac{1}{2}$ in. New York, John Wiley & Sons, Inc., 1961. Price, $5.95.

This book is unique in that it is the first and only book presently available in its field. It is written in two parts, the first part dealing with the consulting engineer's professional relationships to his client, to other consultants, and to the public. The second part deals with internal problems of a consulting practice—the many areas of organization, personnel, plant facilities, procedures, and management situations with which the consulting engineer must cope. The author places emphasis on the problems related to the *operation* of a consulting engineering practice, and *not* on the technical engineering problems involved in construction and design.

PHYSICAL CHEMISTRY, by William F. Sheehan. 618 pages, diagrams, $6\frac{1}{4} \times 9$ in. Boston, Allyn & Bacon, Inc., 1961. Price, $9.95.

This is a new college text written by William Sheehan of the University of Santa Clara's Department of Chemistry. It is presented from a mathematical point of view.

MECHANICAL MEASUREMENTS, by Thomas G. Beckwith and N. Lewis Buck. 559 pages, diagrams, illustrations, 6×9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc., 1961. Price, $8.75.

Designed as a text and reference for both classroom and laboratory sessions at the advanced undergraduate level, this book is an introduction to modern measurement techniques for students of mechanical engineering. Emphasis is placed on the *dynamic*, "mechanics design" aspect of the measurement problem, and its consequent treatment of electrical and electronic methods.

MECHANIZATION OF MOTION, by Lee Harrisberger. 363 pages, diagrams, illustrations, 6×9 in. New York, John Wiley & Sons, Inc., 1961. Price, $8.50.

Primarily intended to present rational procedures for the synthesis of mechanisms, this book combines the fundamentals of kinematics, the techniques of ideation, and new techniques of analysis. This combination is presented in a step-by-step discussion of logical procedures for synthesizing mechanical systems in order to satisfy any motion requirement. The reader is introduced to the process of creating designs which satisfy a realistic need by being given a fresh approach in the teaching of kinematics, one that imparts meaning and purpose to the technology.

THE DESIGN OF SMALL DIRECT-CURRENT MOTORS, by A. F. Puchstein. 407 pages, diagrams, 6×9 in. New York, John Wiley & Sons, Inc., 1961. Price, $12.00.

For use either as a reference or as a textbook, this book is a condensed summary of methods for solving the problems involved in the calculation and design of direct-current machines. The author gives methods both for those cases in which mechanical parts are available and for those in which no mechanical parts are available and design must proceed "from the ground up." Each presentation of a method includes precise instructions, alternate procedures, numerical examples, and exact results. Throughout the book, emphasis is on the electromagnetic aspects of design problems and orderly procedure, and many of the procedures given have not appeared in any other publication.

INFRARED ABSORPTION OF INORGANIC SUBSTANCES, by Katheryn E. Lawson. 200 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1961. Price, $6.75.

An invaluable source book of information on inorganic spectra, this is the first book to compile and review all the currently available material on the infrared absorption spectra of inorganic substances.

Part one summarizes current experimental techniques with reference to the controlling factors, and cites the varied applications of infrared spectroscopic studies of inorganic materials. The second part of the publication presents as many correlations and assignments as are available from the experimental data of various observers. The period from January, 1952 through April, 1960 is covered by a glossary and an extensive annotated bibliography with index cover.

AN INTRODUCTION TO THE PRINCIPLES OF COMMUNICATION, by John C. Hancock. 253 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $9.50.

The basic concepts of non-deterministic or random time functions and their application to the analysis and synthesis of communication systems is discussed in this text, and a clear and simplified approach to these principles is presented so they may be easily understood at the undergraduate level. Some outstanding features of the book are: the frequency versus time domain; an introduction to probability, probability density, the autocorrelation function, spectral density, all emphasizing problem solving; a unified treatment of existing communication systems; and a simplified analysis of noise. The most outstanding single feature of the book is its simplified treatment of topics previously taught at the graduate level. Careful attention has also been given to the integration of traditional topics with recent advances in the field of communication theory. Basic fundamentals are stressed in this text, rather than specific devices or systems. Numerous examples illustrate the application of the theory covered.

THE ENCYCLOPEDIA OF MICROSCOPY, edited by George L. Clark. 693 pages, diagrams, illustrations, 6¾ × 10 in. New York, Reinhold Publishing Corp., 1961. Price, $25.00.

A companion volume to *The Encyclopedia of Spectroscopy*, this text is a unique and authoritative achievement that will become a landmark in its subject. With the rejuvenation of microscopy by discoveries such as the electron, X-ray and ultrasonic microscopes, this comprehensive treatment of 26 kinds of microscopy takes on major significance. The book includes over 140 specially written articles by internationally recognized experts who survey the broad field of microscopy. The biological, medical, pharmaceutical, forensic, chemical, engineering and industrial applications of microscopy are presented. Some outstanding contributions to this enclyclopedia are the coverage of every conceivable aspect of electron, electron mirror and field emission microscopy by the most eminent authorities in the world; and entirely new and unpublished material on the solid image microscope, the ultrasonic microscope, and on electronic combinations such as flying spot and TV microscopes. More than 600 illustrations include actual electron micrographs, photographs of modern equipment, and diagrams.

PARTIAL DIFFERENTIAL EQUATIONS AND CONTINUUM MECHANICS, edited by Rudolph E. Langer. 397 pages, illustrations, 6 × 9 in. Madison, The University of Wisconsin Press, 1961. Price, $5.00.

The nineteen papers comprising this volume were presented at an international conference conducted by the Mathematics Research Center at the University of Wisconsin in June, 1960. In addition, abstracts of 45 papers presented at the conference are included. The nineteen lecturers represented ten countries, and their contributions to the conference are evidence of the broadened viewpoint mathematicians have adopted in examining the potentialities of differential equations.

THEORY OF MARKOV PROCESSES, by E. B. Dynkin, translated by D. E. Brown and edited by T. Kovary. 210 pages, 6 × 8½ in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1961. Price: $11.95 (trade); $8.95 (text).

Mathematicians will welcome this English translation of this Russian work covering both general state spaces and general transition functions, under the theory of Markov processes. After an introductory chapter, the author defines Markov processes (both stationary and equivalent), discusses subprocesses, the construction of Markov processes with given transition functions, covers strictly Markov processes and subprocesses, and treats the conditions for boundedness and continuity of a Markov process. About 30 pages of supplementary notes, references, and several indices complete the book.

ELEMENTS OF NUCLEAR ENGINEERING, by Glenn Murphy. 209 pages, diagrams, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1961. Price, $7.50.

Intended as a text for a survey course in nuclear engineering, this book presents, at the college senior level, the scope, potentialities, and limitations of nuclear engineering. Its aim is to help the student decide whether he wishes to continue in the field, as well as to give him an idea of the various specialties he may choose. The text is divided into three sections: background (concepts and principles of nuclear engineering); a presentation of the reactor core and how it works; and a survey of radiation, with applications.

ATOMIC ENERGY WASTE—ITS NATURE, USE AND DISPOSAL, edited by E. Glueckauf. 420 pages, diagrams, plates, 6 × 9¾ in. New York, Interscience Publishers, Inc.; London, Butterworth & Co., Ltd.; 1961. Price, $14.00.

The great strides in the field of atomic energy waste disposal are evidenced in this volume covering such materials. Seventeen papers by specialists cover six broad fields: the waste material itself; the effects of radiation on materials and living organisms; the treatment of radioactive wastes; fission products as sources of radiation; and the uses of radiation in research and agriculture. The book is a valuable record of what has already been done, as well as a guidepost to what may be accomplished in the future.

CREATIVE ENGINEERING ANALYSIS, by Frederick L. Ryder. 281 pages, diagrams, 6 × 9 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1961. Price: $9.00 (trade); $6.75 (text).

This book has been prepared to teach the technique of solving engineering problems rather than subject matter. The problems are those not directly associated with applicable formulas and are, therefore, difficult to analyze. Fields covered in this unusual book are mechanics, strength of materials, electricity and electronics, and miscellaneous subjects such as servomechanisms, steam ovens, heat loss, and gamma ray shielding. A lengthy chapter on aids to problem solving in general covers notation, inductive analysis, units, energy methods, error analysis, mathematical aids and many other useful tools for solving engineering problems.

GENERAL RELATIVITY AND GRAVITATIONAL WAVES, by J. Weber. Interscience Tracts on Physics and Astronomy No. 10. 200 pages, diagrams, 5 × 8 in. New York, Interscience Publishers, Inc., 1961. Price: $4.50 (cloth); $2.50 (paper).

A revived interest in general relativity has made this new tract of use particularly in quantum field theory and in the new approach to gravitation. Nine chapters cover the equivalence principle, generalization of the special theory of relativity, Riemannian geometry and tensor calculus, field equations of general relativity and electromagnetism, experimental tests of general relativity, the conservation laws, gravitational waves, detection and generation of gravitational waves, and selected topics in general relativity.

EXTRACTIVE METALLURGY OF COPPER, NICKEL, AND COBALT, edited by Paul Queneau. 647 pages, illustrations, 6 × 9 in. New York, Interscience Publishers, Inc., 1961. Price, $22.50.

The twenty-four papers included in this volume were presented at an international symposium held in New York in February 1960 and sponsored by The Extractive Metal-

lurgy Division of The Metallurgical Society. The papers were divided into five areas: general (including fluid bed roasting, descriptions of plants, processes for separation and the role of corrosion in choosing materials); fuel-fired smelting and converting; electric furnace smelting; atmospheric and elevated pressure leaching; and refining. An extensive bibliography completes the volume. These proceedings should serve as a stimulus to metallurgists to probe more deeply into the possibilities of increasing the usefulness of these materials.

PROGRESS IN SEMICONDUCTORS, VOL. 5, edited by A. F. Gibson, F. A. Kröger and R. E. Burgess. 316 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1961. Price, $11.00.

The seven papers selected for inclusion in this fifth volume reviewing the field of semiconductors are: The Electrical Properties of Semiconductor Surfaces, by T. B. Watkins; The Absorption Edge Spectrum of Semiconductors, by T. P. McLean; The Chemical Bond in Semiconductors, by E. Mooser and W. B. Pearson; Thermal Conductivity of Semiconductors, by J. Appel; Indium Antimonide, by T. S. Moss; Magneto-optical Phenomena in Semiconductors, by B. Lax and S. Zwerdling; and The Band Structure and Electronic Properties of Graphite Crystals, by R. R. Haering and S. Mrozowski.

GALAXIES, by Harlow Shapley. Revised edition, 186 pages, illustrations, 6 × 9 in. Cambridge, Harvard University Press, 1961. Price, $5.00.

In 1943 Dr. Harlow Shapley, at that time Director of the Harvard College Observatory, published his book *Galaxies*. Today, almost 20 years later, the revised edition has come off the press. Because of the almost fantastic advances in astronomy and the tools of the astronomer, there has occurred a change in our concept of distances and composition of these stellar structures. As a point of fact in the first edition we were told that with the 100-in. Hooker Telescope we could penetrate space to a depth of 1.6 billion light years. Today, with the 200-in. Hale Telescope we can probe to a distance of 10 billion light years.

KERNENERGIE-TECHNIK, by H. Engel and K. O. Thielheim. 299 pages, 5¾ × 8 in. München, Verlag Moderne Industrie, 1960. Price, DM 36.

The authors first review the physical and engineering bases of nuclear energy technology, treating such questions as chain reactions, control, regulation, cooling and shielding of reactors, the production and preparation of fuels, and the monitoring of radiation. The next section deals with special reactor types—heterogeneous water reactors, gas-cooled graphite-moderated, and liquid-metal fuel reactors, as well as fast reactors.

A section on the economics of nuclear energy plants examines the cost structure and future prospects of nuclear energy generation. The authors report on the state of atomic energy planning in West Germany, the Euratom countries, the U. S. A., Great Britain, and the U. S. S. R. A final chapter treats the prospects of nuclear ship propulsion, and energy generation from thermonuclear reactions.

THE FRANKLIN INSTITUTE

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, June 14, 1961.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 14, 1961.

MR. FRANCIS G. TATNALL in the Chair.

The following reports were presented for final action:

No. 3381: Work of James A. Van Allen in Space Sciences.

This report recommended the award of an Elliott Cresson Medal to James Alfred Van Allen, of Iowa City, Iowa, "In consideration of his many contributions and pioneering achievements in the field of space science; in particular, for his discovery of regions of trapped radiation in space, now termed the 'Van Allen Radiation Belts.' "

No. 3393: "The Mössbauer Effect."

This report recommended the award of an Elliott Cresson Medal to Rudolf L. Mössbauer, of Pasadena, California, "For his verification of the special theory of relativity of time due to velocity; for development of an unimaginably sensitive technique for measuring time and penetrating some of its long-kept secrets using relatively simple apparatus; and for his contribution to the interlacing of various branches of physics such as nuclear and solid state physics."

No. 3396: The Franklin Medal.

This report recommended the award of The Franklin Medal to Detlev W. Bronk, of New York, New York, "In recognition of his learned investigations of electrical and biochemical properties of nerves, for his contributions to our understanding of the central and peripheral nerves, for the perfection of his experimental techniques which set an inspiring example for co-workers, and for his scientific leadership in biophysics."

No. 3397: Ballantine Medal.

This report recommended the award of the Stuart Ballantine Medal to Leo Esaki, of Briarcliff Manor, New York, "For his discovery and utilization of quantum mechanical tunneling in semi-conductors and for having pioneered an important new area of electronic investigation."

No. 3398: Ballantine Medal.

This report recommended the award of a Stuart Ballantine Medal each to Nicolaas Bloembergen, of Cambridge, Massachusetts, and H. E. Derrick Scovil, of New Vernon, New Jersey, "For the invention of the three-level solid state maser and its subsequent development for practical use."

No. 3399: Levy Medal.

This report recommended the award of the Louis E. Levy Medal to John H. Holland, of Ann Arbor, Michigan, "In recognition of his outstanding paper, 'Cycles in Logical Nets,' appearing in the September 1960 issue of the Journal of The Franklin Institute."

D. S. FAHRNEY
Secretary to Committee

NEW BOOKS IN THE FRANKLIN INSTITUTE LIBRARY

MATHEMATICS

LECTURES ON DIFFERENTIAL AND INTEGRAL EQUATIONS, by Kōsaku Yoshida. Interscience, 1960.

ADVANCED CALCULCUS, by David V. Widder. 2nd ed. Prentice-Hall, 1961.

AN INTRODUCTION TO LINEAR STATISTICAL MODELS, by Franklin A. Graybill. McGraw-Hill, 1961.

THEORY OF FORMAL SYSTEMS, by Raymond M. Smullyan. Princeton University Press, 1961

PHYSICS

INDEX TO THE LITERATURE ON X-RAY SPECTROGRAPHIC ANALYSIS. American Society for Testing Materials, [1961].

UNDERWATER ACOUSTICS HANDBOOK, by Vernon M. Albers. Pennsylvania State University Press, [1960].

RELATIVISTIC ELECTRON THEORY, by Morris E. Rose. Wiley, [1961].

THE NATURE OF PHYSICAL KNOWLEDGE, edited by Lawrence W. Friedrich, Indiana University Press, [1960].

OPTICAL SPECTROMETRIC MEASUREMENTS OF HIGH TEMPERATURES, edited by Philip J. Dickerman. University of Chicago Press, [1961].

FIELD EMISSION AND FIELD IONIZATION, by Robert Gomer. Harvard University Press, 1961.

MECHANICS, by L. D. Landau and E. M. Lifshitz. Pergamon Press, 1960.

THERMODYNAMICS, by Gilbert Newton Lewis and Merle Randall. 2nd ed. McGraw-Hill, 1961.

KINEMATICS OF NUCLEAR REACTIONS, by A. M. Baldin, V. I. Gol'danskii and I. L. Rozenthal. Pergamon, 1961.

NUCLEAR ENGINEERING, by Gilbert Cahen and Pierre Treille. Allyn & Bacon, 1961.

COMPARATIVE EFFECTS OF RADIATION, edited by Milton Burton, J. S. Kirby-Smith and John L. Magee. Wiley, [1960].

NOTES ON QUANTUM MECHANICS, by Enrico Fermi. University of Chicago Press, [1961].

THE APPLICATION OF GROUP THEORY IN PHYSICS, by Grigorii Ia. Liubarskii. Pergamon Press, 1960.

COLLOQUE INTERNATIONAL DE PHOTOGRAPHIE CORPUSCULAIRE, 2nd, Montreal, 1958, edited by Pierre Demers. Les Presses Universitaires de Montreal, 1959.

THE HALL EFFECT AND RELATED PHENOMENA, by E. H. Putley. Butterworths, 1960.

INTERNATIONAL CONFERENCE ON NUCLEAR STRUCTURE, KINGSTON, 1960, edited by D. A. Bromley and E. W. Vogt. University of Toronto Press, 1960.

PRINCIPLES OF MECHANICS OF SOLIDS AND FUELS, by Hsüan Yeh and Joel I. Abrams. McGraw-Hill, 1960.

DECAY SCHEMES OF RADIOACTIVE NUCLEI, by B. S. Dzhelepov and L. K. Peker. Pergamon Press, 1961.

MOTION AND RELATIVITY, by Leopold Infeld and Jerzy Plebanski. Pergamon Press, [1960].

ELECTRODYNAMICS OF CONTINUOUS MEDIA, by L. D. Landau and E. M. Lifshitz. Pergamon Press, 1960.

CHEMISTRY AND CHEMICAL ENGINEERING

ELECTROSTATIC SEPARATION OF MIXED GRANULAR SOLIDS, by Oliver C. Ralston. Elsevier, 1961.

A PRIMER OF SPACE MEDICINE, by Martin P. Lansberg. Elsevier, 1960.

SYMPOSIUM ON SALINE WATER CONVERSION, CLEVELAND, 1960. American Chemical Society, 1960.

TABLES FOR PETROLEUM GAS/OXYGEN FLAMES, COMBUSTION PRODUCTS, AND THERMODYNAMIC PROPERTIES, by I. I. Berenblut and Anne B. Downes. Oxford University Press, 1960.

KINETICS AND MECHANISM, by Arthur A. Frost and Ralph G. Pearson. 2nd ed. Wiley, [1961].

CHEMICAL PROPERTIES AND IDENTIFICATION OF IONS, by Omer K. Whipple. Wadsworth Pub. Co., [1961].

INDUSTRIAL ORGANIC NITROGEN COMPOUNDS, by Melvin J. Astle. Reinhold Pub. Corp., [1961].

SPECTROCHEMICAL ANALYSIS, by Louis Herman Ahrens and S. R. Taylor. 2nd ed. Addison-Wesley, [1961].

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Today, more than ever before in its 138-year history, there is vital need for The Franklin Institute effectively to promote education in science and technology. It is imperative that we meet this challenge by providing adequate educational opportunities in these fields. This requires vision, objective planning, and money. We have more than enough of the first two requisites, but far too little of the third.

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CURRENT TOPICS

Electronic Aid to Music Composers. —Scientists of the Radio Corporation of America have developed an experimental electronic system that can help a composer to create new music by suggesting variations and new tone combinations based on his original ideas. The system has also been used as a tool in the broader communications studies.

The system was described to the National Academy of Sciences by Dr. Harry F. Olson, Director of the Acoustical and Electromechanical Research Laboratory at RCA's David Sarnoff Research Center in Princeton, N. J. He said the equipment was developed as part of a general RCA Laboratories study over the past several years of communications theory, especially in relation to sounds and music. Other phases of the same program have resulted in the electronic music synthesizer and the experimental phonetic typewriter.

In experiments at RCA Laboratories, the unit has demonstrated its prowess by producing original tunes in the style of Stephen Foster, guided only by circuit arrangements tailored to produce variations based on the melodic and rhythmic characteristics of Foster's music. Dr. Olson said that the circuits could be easily rearranged to provide variations based on any style of music with which a composer wished to work, including unconventional scales and rhythms.

The experimental unit, developed by Dr. Olson and Herbert Belar, Fellow of the RCA Laboratories technical staff, is a special-purpose type of computer known as a "random probability" system. This was defined by Dr. Olson as an arrangement of circuits designed to select notes in random fashion from among many choices, with the probability of the choice determined by the frequency with which various note sequences occur in the style favored by the composer. He pointed out that the composer sets the pattern of the controlling circuits in advance, using the system as a tool which ranges over the many possible variations that he may choose to develop his basic theme.

"In this way, the system can provide the composer with a substantial aid in his creative work," said Dr. Olson. "It cannot substitute for the composer in producing entirely original music, but it can furnish him with a useful and versatile electronic tool for his use in developing his own original ideas."

He added that the principles used in the system can be applied not only in musical composition, but just as readily in many other activities in which the objective is to develop variations on basic ideas. As examples, he mentioned the styling of automobiles, television sets, or other products with a definite and limited range of characteristics and functions, as well as the selection of color combinations, or sequences of words.

"The machine will generate a tremendous quantity of such variations," he said. "The operator—whether he is a composer, an artist, or a designer —can study the output and use his judgment in selecting the variations that fit his plan."

As described by Dr. Olson, the sys-

tem is composed of tubes, relay switches, and associated circuitry which could fit into a cabinet no larger than a standard office desk. Major elements in the system are the sets of relay switches, interconnected in a manner that provides a maximum number of possible paths that can be followed by signals passing through the system, just as floating sticks might follow different channels in drifting through a river delta with many branching streams. The sequence of operation in the relays is determined within any desired limits by an array of wire connections that can be changed readily in the manner of connections on a telephone switchboard.

By arranging the wire connections, the composer can limit the possible courses of signals through the relays—and thus the range of notes that the system will select in producing variations—to fit his basic idea. Thus, if he tends to create music in which notes AB are followed by notes C, F, or G because he favors this sequence in his individual style of composition, he will set the connections in such a way that the relays will arbitrarily produce one of the three desired notes, and avoid the others that he does not want. If he prefers one of the three notes more than the others, he can arrange the connections so that the preferred note will be produced more frequently in the flow of variations.

Dr. Olson explained that the use of the system as a composing aid requires related equipment to translate the output of signals into audible tones for recording and study by the composer. In the laboratory experiments, he said, the system was connected to an electronic music synthesizer, designed to convert electrical signals into musical sounds.

Discussing other possible applications, Dr. Olson explained that the electrical output of the system could be converted to numbers corresponding to colors on a chart for use in studying variations of color composition, or to codes referring to various design features in studying variations of styling.

Mammouth Saucer-Shaped Antenna to Aid Global Communications.—A unique radio research facility, employing a 120-ft.-wide saucer-shaped antenna with a surface contour of unprecedented accuracy, is being built for the U. S. Air Force in Tyngsboro, Mass., for global communications and space studies.

Construction of the $3,800,000 research facility, which is scheduled to be "on the air" by the end of 1962, is managed by the Electronic Systems Division at Hanscom Field, one of four divisions of the Air Force Systems Command.

Lincoln Laboratory of Massachuetts Institute of Technology is playsing a major role in its design and construction, and upon its completion, will operate and evaluate the facility on behalf of the Air Force.

The research instrument will sit upon Haystack Hill in Tyngsboro, a short distance from Lincoln Laboratory's famed Millstone Hill radar in Westford, forerunner of today's high-power, long-range radars and one of the Free World's prime sources of satellite tracking information.

The extremely precise antenna, whose surface must never vary more than a 0.076 in. over its entire quarter-acre surface, will be completely sheltered from New England's temperamental weather by a new type of spherical radome 150 ft. in diameter.

The radio facility is designed to serve as a test-bed for development of the large ground-based transmit-

ting and receiving equipment that will be needed to operate high-capacity satellite-relay systems for round-the-world communications. It features a new "pluggable" electronics package for unparalleled flexibility of operation.

This new Air Force research antenna will be the most precise structure of its kind ever attempted for an antenna of this size. The high precision is necessary to concentrate the transmitter power into a needle-sharp beam of very high intensity, and to provide an equally sharp receiving beamwidth to discriminate weak, distant sources of radio signals.

At an operating frequency of ten thousand megacycles per second (10,000,000,000 cycles per second), the width of the Haystack beam will by only one twentieth of one degree. The beamwidth of the Millstone Hill radar is about two degrees, some forty times broader.

As an instrument for radio communications research, the Haystack system will be used to probe the troposphere and the ionosphere, and to study atmospheric irregularities that may limit radio antenna performance (how narrow a beam can actually be produced, and how accurately it can be aimed), similar in principle to the limits the atmosphere imposes on astronomical observations with large optical telescopes.

It will also be used to measure electrical noise generated in the atmosphere, and to study the diffraction (bending) and scattering of radio waves at the super high frequencies that will be used for global satellite-relay communication systems now under development by various agencies.

The Haystack antenna could also be used as a part of a radar, although no plans for such use have been established. As a radar it could also be used to obtain valuable new information about the surface of the moon and the nearest planets, especially Venus whose surface is perpetually hidden from view by a heavy veil of clouds, by bouncing a signal off that planet's surface.

To maintain the extremely close tolerances demanded by this precise instrument, the antenna will be completely enclosed in a Fiberglas and aluminum metal-space-frame radome. Without this protection, wind pressure and the sun's heat on the quarter-acre reflector surface would distort it beyond the allowable maximum of 0.076 in.

Surface of the main reflector consists of 96 panels of $\frac{1}{2}$-in. thick aluminum honeycomb. The panels will be held in place by five concentric rings interconnected by tension rods which will be tuned like piano strings for exact tension adjustment. This structure provides the greatest possible rigidity with the smallest possible weight.

The Haystack Hill radome will be 150 ft. across at its widest point, 134 ft. high, and 90 ft. across its base. Frame of the radome will be of hollow aluminum beams up to 15 ft. long. The glass fiber reinforced plastic membrane panels mounted on this framework will be only one thirty-second of an inch thick.

An important new feature of the Haystack Hill system is the large "package" that contains all the radio equipment, including both the transmitter and receiver. Although the power supply is located on the ground, this package is mounted on the antenna system, just in back of the center of the big dish, and moves with it.

Eight feet square and 10 ft. long, this package can hold 2 tons of elec-

tronic equipment for a particular experiment. It can be quickly "unplugged" and another unit substituted for a different experiment. Since changes that might ordinarily take weeks or months with other radio research instruments can be accomplished in a few hours, this gives extraordinary flexibility to the equipment. The "pluggable" package allows the Haystack Hill facility to be used over a wide range of frequencies and for a variety of different experiments without costly and time-consuming modifications.

In addition, this arrangement improves performance by eliminating long radio-frequency transmission paths. Long transmission paths can be harmful in two ways: they can cut down the transmitted power by introducing unavoidable losses at high power levels, and they can introduce noise that will impair the sensitivity of the receiver and its capability to detect faint incoming signals.

In operation, radio energy will be beamed from the electronics package through the center of the 120-ft. dish toward a secondary reflector mounted close to the focus of the main dish. Radio energy will bounce back from the smaller, 9-ft. dish to the large antenna, which then beams it in the desired direction.

ESD is building the Haystack Hill facility as part of its continuing research into advanced methods of communications to meet the needs of aerospace operations. The division is charged with the responsibility of developing and testing complex electronic systems which will help military commanders obtain vital information for making decisions on the best way to employ their forces and which will get the commander's decision to his forces in a rapid and reliable manner.

Shielding with Electrical Energy?—Shielding a space vehicle with electrical energy as the earth is protected by a belt of electromagnetism may eventually be the most practical way to save man from lethal radiation in space travel.

General Electric scientists believe a light weight system using electrical energy can be invented. If found, such a system would reduce the heavy weight penalties of the current method of using shielding materials.

A thorough year-long investigation into the possibility of using an electrical shielding system is now under way at the Space Sciences Laboratory of the Company's Missile and Space Vehicle Department under a $75,000 contract with the National Aeronautics and Space Administration. Besides studying the possible use of electrical energy as shielding, the NASA contract also calls for an investigation of other possible methods, such as using lead in combination with other materials and other materials alone or in combination.

People on this earth are protected from space radiation because of the magnetic field created by the North and South poles. This magnetic field deflects the deadly rays and changes their direction so that only minute amounts of radiation penetrate the earth's atmosphere.

Creating a similar protective electrical shield around a space vehicle is possible now with existing equipment, but the scientists say that the weight of this equipment would be even more prohibitive than lead. However, they believe of the many possible approaches to using an electrical shield at least one may provide the desired weight reduction, while there are at present only limited possible ways of using lead and other effective mate-

rials, and these seem to offer little hope for appreciable savings in weight.

The need for finding improved shielding systems if man is ever going to travel far in space is apparent in the current calculation that the equivalent of 50 pounds per square foot of water is now believed to be needed to protect man from a lethal combination of cosmic rays, the earth's radiation belt, and solar flares, scientists point out.

A comprehensive study of possible shielding methods has never been performed because the prohibitive danger of space radiation has only recently been evaluated, the General Electric scientists said. From their study, they hope to find a system or systems for later practical application.

Progress in Project Mohole.—A French drilling and coring device, believed never to have been previously used in the United States, was successfully tested in the final operations of the experimental drilling program for Project Mohole near Guadalupe Island. The drilling took place in 11,700 ft. of water off the west coast of Mexico.

The device, known as a turbodrill, rotated a diamond-studded bit at high speeds for 2 hr. through dense basaltic rock more than 500 ft. below the ocean floor. Unlike the customary drilling bit, which is rotated at relatively slow speeds by the turning drill pipe, the turbodrill was rotated independently at a speed of 750 rpm. by sea water pumped down the stationary drill pipe at 1750 psi.

By means of a wire line retrieval system, two small cores of the basaltic and overlying material were retracted through the drill pipe to the deck of the drilling barge for examination.

The turbodrill, manufactured by Neyrtic, of Grenoble, France, was operated on the *CUSS I* drilling barge under the supervision of Guy Bourdessoule.

Mohole scientists have reported fossil evidence of a flowering of sea life in the area roughly 25 million years ago. More than 100 ft. of nearly continuous core of the deep ocean ooze showed that sea life in the Guadalupe Island area was prolific for some seven million years, but that now the area is an oceanic desert in comparison.

The findings were announced by the National Academy of Sciences–National Research Council, and the National Science Foundation, who also reported the first heat measurements ever made 500 ft. below the ocean's floor. Temperature readings at that level obtained by the *CUSS I* were reported as somewhat higher than had been expected on the basis of earlier ocean bottom measurements.

The *CUSS I* is carrying out the first deep-sea drilling in history to gather data and experience for further planning of Project Mohole, a plan to drill into the boundary between the crust of the earth and its mantle. The drilling program is under the guidance of the Academy–Research Council, with primary support by the Foundation.

Preliminary studies of the 109 feet of soft sediment core have been completed by Chief Scientist William R. Riedel and his associates aboard the research vessel Spencer F. Baird, of Scripps Institution of Oceanography, University of California. The studies indicate that the gray-green material formerly described as a clay is more properly called an ooze, since 80 percent of it consists of microscopic skeletons of plants and animals.

The 500 feet of sediment was determined to be upper Miocene in geologic time on the basis of a correlation of the fossils with similar ones of

known age found in continental rocks. The rate of deposit of sediments at the site was estimated as about two centimeters of ooze per thousand years. Material deposited near the surface contains few organic remains, indicating less abundant marine life in recent times. The continuous sediments record is believed to be of value to the work of geologists ashore in that it may be used to link the intermittent fossil groups of the Miocene found in rocks on land. The descriptions of the cored material were made by Dr. Harry Ladd, geologist of the U. S. Geological Survey, Department of Interior.

The Guadalupe Island area, in 12,250 ft. of water, off the western coast of Mexico, was previously known to have a higher than average heat flow, or transfer of heat by conduction upward through the crustal rocks from the molten interior. This heat flow previously could be measured only by comparing the temperature of the ocean bottom with that of the sediments that could be reached with a probe—about 50 ft.

The first direct measurements of the deeper sediments confirmed the high heat flow previously expected, but indicated even higher readings than had been extrapolated for these levels.

The ship obtained temperature measurements of 44.6 F. (7 C.) at 140 ft. below the bottom, and 75.2 F. (24 C.) at about 500 ft.

The temperature work is being done by Dr. Richard P. von Herzen, research geophysicist of Scripps; Dr. Arthur Maxwell, oceanographer of the Office of Naval Research; and Dr. Jack I. McLelland, chief engineer for the drilling program.

Geophones and seismic wave detectors were lowered into the hole to measure the velocity of sound in the deep rocks and to redetermine the thickness of the various layers beneath the reach of the drill. The auxiliary vessel Orca, of Scripps, fired 50-pound charges underwater some distance from the hole, and the incoming sound waves were recorded. Preliminary analysis of the readings indicate that sound waves penetrate the soft sediment at a rate of 1.59 kilometers a second, a rate considerably less than the value generally assumed by seismologists. If this low velocity generally exists, oceanographic sediments could be thinner than has heretofore been supposed.

The seismic soundings were conducted by Dr. Russell W. Raitt, and Dr. George G. Shor, Jr., both Scripps research geophysicists.

What are believed to be the first fixed-position current measurements in the deep ocean were reported by Robert M. Snyder, staff engineer for instrumentation, a staff member of Woods Hole Oceanographic Institution, Woods Hole, Mass.

Measurements of current velocity for as long as 9.5 hours at 1,500 feet below the surface were made with a rotor-type meter suspended between the *CUSS I* and one of the deep-moored buoys.

Current velocities near the surface ranged from 0.3 to 0.6 knot in continually changing directions, as though a complicated pattern of eddies was superimposed on a general drift of the water to the northeast. The same effect was observed for 1,500 ft., where the velocities were 0.2 knot.

University to Use Computer for Paperwork, Medical, and Fisheries Data.—An electronic data-processing system that can run the performance gamut from student registration and accounting to medical statistics and up-to-the-minute information on commercial fishing in Florida waters will

be installed this fall by the Radio Corporation of America at the University of Miami.

Floyd Howell, Jr., Director of the University's Machine Accounting and Computer Center Department, said use of an RCA 301 system will lead to tighter budget control and faster processing of reports of 13,000 students attending the University's four yearly semesters.

He pointed out that at present there is a lag of five to seven days on budget reports because of computations involved, and estimated that, with the new equipment, the delay would be cut to two or three days.

The RCA 301 is a compact and fully-transistorized system designed for medium-scale operations. The system will be used by a Department of Biometry and Statistics to be established by the Medical School. Studies will be made on gynecology, tumor registry, epidemiology, neurology, polio and other medical subjects. Voluminous statistics on patients in the university hospital will be recorded for computer analysis.

For instance, a study of premature births will entail the evaluation of such factors as the mother's age, weight, prenatal care, economic status, family and medical history, complications and other pertinent details.

The computer also will be used in psychological testing conducted by the university's Guidance Center in student counselling and placement work.

An analysis of commercial fishing statistics will be made for the State Department of Fisheries. Information will be developed at regular intervals on the types and quantities of fish brought in by fishermen of the various counties, the type and amount of gear used, numbers of fishermen per boat, how long the boat is out and other factors of value for over-all industry reports, evaluation and projections.

World-Wide Aerosol Layer.—A world-wide aerosol layer consisting mainly of sulfur particles has been discovered by the Geophysics Research Directorate of the Air Force Cambridge Research Laboratories. The layer completely envelops the earth, forming a 3-mile thick shell about 11 miles out in space.

The GRD work provides the first direct evidence for such an aerosol layer, previously proposed to explain certain twilight effects. Its existence was first indicated to GRD scientists by air samples collected in balloon and aircraft studies of fallout distribution. In analyzing the samples, Dr. C. E. Junge, C. W. Chagnon, and J. E. Manson noted that the concentration of sulfur was exceptionally large between altitudes of 11 and 14 miles. In an area extending from Texas north to Alaska and south to Australia, experimentation has shown that this concentration remains remarkably uniform at various times and latitudes. The layer has thus been established as a persistent world-wide phenomenon.

Detailed studies were initiated over a year ago and are continuing. They show that the concentration of particles within the layer is about 16 particles per cubic inch—three times the number at adjacent levels. These particles are between 0.1 and 2 microns (1 micron = 0.000039 in.) in size. In other words, some five hundred of these particles would be about the same size as a single red blood cell.

Electron microprobe analyses of samples collected by balloons and aircraft showed that the layer is about 85 percent sulfur. Silicon, iron, and

aluminum were also found in some samples. This analysis, along with a determination of particle hygroscopicity (absorbency) and an electron diffraction study, indicates that the particles are ammonium sulfate. The iron and silicon content of some samples may be explained by a temporary fluctuation in micrometeorite influx.

The aerosol layer is thought to originate at its observed location. Probably gases such as hydrogen sulfide or sulfur dioxide, known to be present at lower altitudes, enter the stratosphere and are chemically changed either by sunlight or ozone.

The world-wide aerosol layer may at last provide an explanation for the so-called Purple Light. This conspicuous red disk of light is frequently seen above the point of sunset or sunrise. A thick aerosol layer in the stratosphere could scatter light in such a way to produce just this phenomenon.

Aluminum Tube-In-Strip Simplifies Fabrication Space Simulation Chambers.—Fabrication of General Electric's new space simulation chambers —which will permit decisions about spacecraft designs and thermal balance problems without actual flights —has been greatly simplified by the use of aluminum Tube-In-Strip made by Revere Copper and Brass Incorporated, according to Charles B. Hood of CryoVac, Inc., Columbus, Ohio.

Revere aluminum Tube-In-Strip, consisting of one piece solid sheets or strips in which inflatable tubing is integral with the metal, is being used in two distinct ways in the simulation chambers being built for installation at the General Electric Missile and Space Vehicle Laboratory at Valley Forge, Pennsylvania, Mr. Hood explained.

In one use an internal shell of Tube-In-Strip, anodized black and with liquid nitrogen pumped through the tubes, will appear to the test object like the cold, dark regions of outer space. The test object, or satellite, will be completely surrounded by this shell, sometimes called the heat sink.

The second use will be in a number of small panels, called cryopanels, which will be attached to the interior of the shell. These small Tube-In-Strip panels will be cooled to −425 F. with helium gas supplied from an external refrigerator. At this extremely low temperature the vapor pressure of air is less than one trillionth of an atmosphere. As a result the air in the chamber will freeze out on the panels, creating the extremely high vacuum conditions of outer space.

Mr. Hood declared that Revere aluminum Tube-In-Strip was chosen for these applications to eliminate the problem of bonding long lengths of tubing to metal sheet by soldering or welding.

"The integral tube, which is a part of the original sheet, provides the best possible heat transfer characteristics in this type of application," he said. "The integral tube, which is essentially seamless, will also be free of small leaks, which is extremely important in high vacuum work where even the smallest leakage can reduce the vacuum attained."

New Graft Cellulose.—A new industrial raw material resulting from the highly uniform grafting of an organic compound to the cellulose polymer and representing a significant advance in cellulose chemistry has been introduced by the Eastern Research Division of Rayonier Incorporated.

Trade-named "Ethylose," the new material, an outgrowth of the chemical

cellulose producer's graft polymer program, is seen by company officials as only the first in a whole family of modified celluloses in which various chemicals and polymers are grafted onto a basic cellulose backbone.

The company is hopeful that the new material will broaden and further diversify the uses for cellulose, a readily available and easily replenishable low-cost chemical from trees. Potential markets of special interest include coated papers, textiles, paints, ceramics and foods. Rayonier Research said it foresees such immediate new products as textile sizes, binders and coatings.

Dr. R. L. Mitchell, general manager of research, explained that "Ethylose" is a high-quality, purified wood cellulose modified or grafted with a small amount of ethylene oxide to alter its behavior characteristics.

He stressed that the material is insoluble in water. But it can readily be dissolved in aqueous alkali to render stable and useful solutions.

The amount of substitution of ethylene oxide is only 4 per cent, Dr. Mitchell declared, adding that "this is the first time an inherently soluble cellulosic bulk raw material has been commercially produced at such a low level of substitution."

It was noted that "Ethylose" differs from viscose in that it is stable in the small pellet form in which it is prepared as well as in solution. It can be shipped for long distances or stored for long periods without deterioration and without the need and expense of handling large quantities of water and other solvents.

"Ethylose" solutions can be adjusted to proper solids-viscosity ranges for coating, tub sizing or surface spraying. "Ethylose" can be regenerated by alkali removal or heat followed by washing. The particular method selected will depend on the process and end-use requirements, Dr. Smith said. Neutralization can be accomplished by acid-salt solutions such as 10% H_2SO_4 and 15% Na_2SO_4, or by acid alone. Acid-salt solutions of a dehydrating nature regenerate a product with higher density, lower gel swelling and best physical strength, he said. Temperatures in the range of 40–60 C. are preferred.

Ceramic Tubes Relayed Physical Data on Astronaut.—Tiny ceramic tubes not much bigger than vitamin capsules flashed back to earth the heart throbs of America's first astronaut. As he hurtled through space aboard the Project Mercury capsule, the astronaut underwent a thorough and continuous physical checkup to enable doctors and scientists to study the effects of the trip. Signals produced by the astronaut's heart beat and other body processes were amplified by three miniature ceramic vacuum tubes so they were strong enough to travel back to radio receivers at tracking stations.

Special sensors attached to the space man's body picked up his heart beats, pulse rate, breathing and temperature. They were relayed through wires built into his pressurized space suit, to a miniature telemetry transmitter which contains the tubes.

The tubes make up the power amplifier stages of the VHF transmitter where they boost the strength of the signals to 3 or 4 watts—plenty of power to cover the distance. A single G-E type 7720 drives two 7296's in parallel to produce the output signal.

The transmitter itself is only slightly larger than two packages of cigarettes and weighs less than a pound.

In addition to the running physical exam, the transmitter also told scientists back at the tracking station what the weather was like inside the capsule, that is, air temperature, pressure, force of gravity, etc.

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